The only way to learn the material well (and thus receive a good grade) is by solving many problems, and struggling to solve the more challenging ones. Unfortunately, there is no shortcut.

Please read all the relevant sections in the textbook.

Homework problems will be assigned bi-weekly. Please only submit highlighted problems for grading. Please hand the problems to me before the class begins. You may discuss these problems with other students, but you must independently write up and submit your own solutions. Copying any part of a solution from a book, solutions guide, or website is cheating! Students are expected to abide by the Georgia Tech Academic Honor Code.

Late homeworks will not be excepted, but you may drop your two lowest homework grades. Emailed homeworks will only be accepted with prior agreement of the Instructor - else they will not be accepted.

To receive credit for a solution, you must clearly indicate how you obtained your solution. You will receive no credit without an explanation. Please keep your written answers brief; be clear and to the point. The grader will deduct points for rambling and for incorrect or irrelevant statements. Please do not show arithmetic and most algebra calculations.

In order to grade as many problems as possible, your submitted problem sets should be printed very neatly in large “font” and stapled. Please do not cross out. Write on only one side of each page, in a single column, and do not use paper that has been torn out of spiral bound notebooks. You may typeset your solutions, but this is not required.

1. Solve the IVP
   \[ x''(t) + x(t) = \cos(t), x(0) = 0, x'(0) = 0 \]
   using both the Method of Undetermined Coefficients and the Method of Variation of Parameters.

2. Solve the IVP
   \[
   \begin{pmatrix}
   \dot{x} \\
   \dot{y}
   \end{pmatrix} = \begin{pmatrix}
   1 & 1 \\
   0 & -2
   \end{pmatrix}
   \begin{pmatrix}
   x \\
   y
   \end{pmatrix} + \begin{pmatrix}
   \exp(-2t) \\
   0
   \end{pmatrix},
   \begin{pmatrix}
   x(0) \\
   y(0)
   \end{pmatrix} = \begin{pmatrix}
   1 \\
   -1
   \end{pmatrix}.
   \]

3. Solve the IVP
   \[
   \begin{pmatrix}
   \dot{x} \\
   \dot{y}
   \end{pmatrix} = \begin{pmatrix}
   0 & -1 \\
   4 & 0
   \end{pmatrix}
   \begin{pmatrix}
   x \\
   y
   \end{pmatrix} + \begin{pmatrix}
   1 \\
   \exp(t)
   \end{pmatrix},
   \begin{pmatrix}
   x(0) \\
   y(0)
   \end{pmatrix} = \begin{pmatrix}
   100 \\
   50
   \end{pmatrix}.
   \]

4. Find the general solution
   \[
   \begin{pmatrix}
   \dot{x} \\
   \dot{y}
   \end{pmatrix} = \begin{pmatrix}
   -5 & 1 \\
   4 & -2
   \end{pmatrix}
   \begin{pmatrix}
   x \\
   y
   \end{pmatrix} + \exp(2t) \begin{pmatrix}
   6 \\
   -1
   \end{pmatrix}.
   \]

5. Solve \( y'' - 2y' - 3y = 3t^2 + 4t - 5 \).
6. Solve \( y'' - 2y' - 3y = 5e^{3t} \).
7. Solve \( y'' - 2y' - 3y = t^3 e^{5t} \cos(3t) \).
8. Solve \( y'' - 2y' - 3y = e^{2t} + 3t^2 + 4t - 5 + 5 \cos(2t) \).
9. All problems contained in handout on numerical analysis.
10. When last we heard from Romeo and Juliet, they were involved in a love affair modeled by the 2D ODE $\dot{R} = J$ and $\dot{J} = -R$. Recall that Romeo loves Juliet more the more she loves him but that Juliet only starts to like Romeo if he’s disinterested in her.

We learned that Romeo and Juliet are trapped in a never ending ”make-up/break-up" scenario. Romeo’s best friend, Mercutio, thinks that Romeo is too good for Juliet, however, and has decided to try get them to break up for good. And so he has started telling Romeo how awful Juliet is. Romeo trusts Mercutio and so his ardor for Juliet wanes a bit when Mercutio tells him such things, though he still really likes Juliet. On the other hand, Juliet really hates Mercutio’s guts and the more he disapproves of her relationship with Romeo, the more she likes Romeo!

Let $R(t) =$ Romeo’s love/hate for Juliet at time $t$, $J(t) =$ Juliet’s love/hate for Romeo at time $t$, and $M(t) =$ Mercutio’s disapproval of Romeo and Juliet’s relationship at time $t$. Positive values of $R$, $J$ signify love, negative values signify hate. A positive value of $M(t)$ signifies disapproval. Then a model for this complicated saga is:

$$\dot{R} = J - 2M$$
$$\dot{J} = -R + 4M$$
$$\dot{M} = R + 4J$$

Describe what happens to Romeo and Juliet’s relationship now. In particular, does Mercutio’s tampering have the effect he wants? And do Romeo and Juliet still oscillate between love and hate.