The only way to learn the material well (and thus receive a good grade) is by solving many problems, and struggling to solve the more challenging ones. Unfortunately, there is no shortcut.

Please read all the relevant sections in the textbook.

Homework problems will be assigned bi-weekly. Please only submit highlighted problems for grading. Please hand the problems to me before the class begins. You may discuss these problems with other students, but you must independently write up and submit your own solutions. Copying any part of a solution from a book, solutions guide, or website is cheating! Students are expected to abide by the Georgia Tech Academic Honor Code.

Late homeworks will not be excepted, but you may drop your two lowest homework grades. Emailed homeworks will only be accepted with prior agreement of the Instructor - else they will not be accepted.

To receive credit for a solution, you must clearly indicate how you obtained your solution. You will receive no credit without an explanation. Please keep your written answers brief; be clear and to the point. The grader will deduct points for rambling and for incorrect or irrelevant statements. Please do not show arithmetic and most algebra calculations.

In order to grade as many problems as possible, your submitted problem sets should be printed very neatly in large “font” and stapled. Please do not cross out. Write on only one side of each page, in a single column, and do not use paper that has been torn out of spiral bound notebooks. You may typeset your solutions, but this is not required.

1. Use Pplane to plot the phase plane of the ODE: \( x' = y + y^2, \ y' = -(1/2)x + (1/5)y - xy + (6/5)y^2. \) Make sure the draw stable and unstable manifolds of all saddles.

2. Consider the ODE \( x'' = x - x^2. \)
   (a) Find and classify the equilibrium points.
   (b) Find a conserved quantity.
   (c) Sketch the phase portrait.

3. Consider the ODE \( x'' + x = a + \epsilon x^2, \) where \( \epsilon \) is a small positive number. Find and classify the equilibrium points.

4. Consider the ODE \( v' = -\sin(\theta) - Dv^2, \ v\theta'' = -\cos(\theta) + v^2. \)
   (a) Suppose \( D = 0. \) Show that \( v^3 - \cos(\theta) \) is a conserved quantity, and sketch the phase plane.
   (b) Sketch as much as you can and sketch as much of the phase plane as possible on the case \( D > 0. \)

5. Sketch the phase portrait for \( r' = r \sin(\theta), \theta' = 1. \)

6. Consider the ODE \( r' = r(1 - r^2) + \mu r \cos(\theta), \theta' = 1. \) Show that for sufficiently small positive \( \mu \) a limit cycle exists.

7. Using the definition, find \( \mathcal{L}[t - 2]^2, \) if it exists. If the Laplace transform exists then find the domain of \( F(s). \)

8. Using the definition, find \( \mathcal{L}[f], \) if it exists. If the Laplace transform exists then find the domain of \( F(s), \) where
   \[
   f(t) = \begin{cases} 
   0 & 0 \leq t < 1 \\
   t - 1 & 0 \leq t \geq 1 
   \end{cases}
   \]

9. Does the function \( \frac{e^{t^2}}{e^{t^2} + 1} \) have a Laplace transform?
10. Use Laplace transforms to solve the IVP \( y'' - 4y' + 9y = t, y(0) = 0, y'(0) = 1 \). (You may use Alpha, Matlab, etc. to compute the partial fraction.)

11. Use Laplace transforms to solve the IVP \( y' + 4y = g(t), y(0) = 2 \), where

\[
g(t) = \begin{cases} 
0 & 0 \leq t < 1 \\
12 & 1 \leq t < 3 \\
0 & t \geq 3
\end{cases}
\]

12. Use Laplace transforms to solve the IVP \( y'' + 3y' + 2y = e^{-t}, y(0) = y'(0) = 0 \). (You may use Alpha, Matlab, etc. to compute the partial fraction.)

13. Use Laplace transforms to solve the IVP \( y'' + 4y = \cos(2t), y(0) = y'(0) = 1 \). (You may use Alpha, Matlab, etc. to compute the partial fraction.)

14. More problems TBA.