basic results on normal forms or higher order averaging are given but are too lengthy to place in this review.

The book under review attempts to give a comprehensive treatment of Hopf's theorem, its generalizations and its applications. In many respects the book is very good. The style of writing is good, it is comprehensive in scope, the proofs are detailed, and there are many worked examples some of which are nontrivial. Indeed, the stated purpose and the strong point of the book is the worked examples.

The book is organized around the computational aspects of the problem and therefore obscures the simplicity of the mathematics. Also the book uses the center manifold theorem to reduce the general problem to the two-dimensional problem. This is unnecessary either from a theoretical viewpoint or from a computational viewpoint. In fact after reading a proof of Hopf's theorem that is sprawled over about fifty pages, one is referred to an appendix where an outline of the proof of the center manifold theorem is given. Thus the proof given is not entirely complete. It would have been better to give one of the short eloquent proofs which uses only the finite dimensional implicit function theorem. The computational method presented is also complicated by the fact that the authors first calculate the equations on the center manifold and then normalize the equations on the center manifold. It would be simpler and easier to program if the full equations were normalized once.

If you have a specific differential equation that you expect exhibits a Hopf bifurcation, then this book will give you the procedure and even a Fortran program to verify the hypothesis. However, it is not the book to read if you wish to study bifurcation theory for its intrinsic beauty.

Kenneth R. Meyer
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Strang's book is a refreshing change from the many rather pedestrian books on linear algebra and matrix theory that glut the market. The writing style is informal, bright, and lively—with even a touch of humor at times. But this is not what makes the book important. It is an important book because of the author's neatly crafted treatment of both the theory and the numerical solution of the central problem of linear algebra—the solution of linear algebraic equations. Theory, applications and the problems of numerical solution are interwoven throughout the book. There are enough applications to keep an applied mathematician, scientist or engineer interested and enough theory to lay a firm foundation for further work. In fact, although the treatment is concrete rather than abstract—\( n \)-tuples and matrices rather than linear spaces and linear transformations—there is enough theory so that a "pure" mathematics major could study the book with profit.

There is nothing unusual in starting a linear algebra book with a discussion of the solution of \( n \) equations in \( n \) unknowns by Gaussian elimination. What is unusual is to show right off that Gaussian elimination is equivalent to the \( LDU \) factorization of a matrix. This is followed shortly by the Cholesky factorization for symmetric matrices. Operation counts are used to show that these factorizations provide efficient algorithms for the numerical solution of systems, and that computation of the inverse matrix is not the way to go. One important case where symmetric matrices arise is illustrated by solving a simple self adjoint boundary value by finite differences. The first chapter closes with some
simple examples illustrating the effect of round off errors, the necessity for partial pivoting, and the existence of those horrible beasts—ill conditioned matrices.

The second chapter is a gem. In it the “four fundamental subspaces” associated with a matrix are developed—the column space, the row space and their orthogonal complements. This provides, at an early stage, a solid understanding of the effect of a matrix acting on a vector. When the relationships between these subspaces and the dimensions of these subspaces are understood, the basic theory about linear systems become clear. For example, it is easy to see that $Ax = b$ has a solution if and only if $b$ is orthogonal to every solution of the transposed homogeneous equation $A^T = 0$—an important fact often omitted in texts. The third chapter is another nice chapter on orthogonal projections and least square solutions of inconsistent systems. Other topics presented here are the Moore–Penrose inverse, the singular value decomposition and the $QR$ factorization. These three chapters give a well rounded treatment of the basic theory of linear equations. It is much more complete, better organized and done much earlier than in most texts.

After the necessary chapter on determinants, eigenvalues and eigenvectors are taken up. These topics are motivated by solving systems of differential and difference equations. Many different applications are discussed ranging from heat flow to factor analysis. The emphasis is on diagonalizable matrices and include the standard results for Hermitian and normal matrices. Not too much is done with matrices that are not diagonalizable—there are only three pages of text and an appendix on the Jordan form. Chapter 6 deals with positive definite matrices, the Raleigh and minimax principles, and even includes an introduction to the finite element method.

The problems of numerical computation are always kept in view throughout the book, however in Chapter 7 these matters are discussed in some detail. Included here are the condition number, the power method for finding eigenvalues, the $QR$ algorithm, and iterative methods for solving $Ax = b$. The last chapter provides an excellent introduction to linear programming and game theory. There are three appendices: linear transformations, the Jordan form and computer codes for solving square systems (given in both Fortran and Basic).

There are a few matters which, in this reviewer’s opinion, detract from the overall excellent quality of the book. Sometimes the informal writing style is take a bit too far. Definitions and statements of results or theorems can be hidden in the general discussion. At times students could find it difficult to determine whether something is stated as a fact to be believed without proof or if the proof is being given in an informal discussion or if the proof is being deferred until later. The careful labeling of proofs and use of the “Halmos” symbol would have helped. The treatment of $n$-dimensional geometry could be done more carefully. For instance, in the discussion of Schwarz’s inequality, the author develops the cosine of the angle between two vectors in the plane and then claims that the fact that the cosine is in magnitude less than one “proves” the Schwarz inequality in $n$-space.

In Chapter 5, the author does not show that the differential equation $\dot{x} = Ax$, $x(0) = x$ has a unique solution, yet uses this fact to obtain an expression for the matrix function $e^{At}$. Regarding $e^{At}$, the unfortunate impression is left that the Jordan form is needed to handle the nondiagonalizable case. This is not true. Functions of a matrix can, and probably should, be developed without the Jordan form (see for example Gantmacher’s book). The main tool that is needed is the Cayley–Hamilton theorem which, by the way, is only mentioned in one problem.

One final point. In his preface the author states that his aim is to teach basic linear algebra (which is certainly accomplished) and asks mathematicians who might want to use his book not to be put off by discussions of operation counts and other remarks on numerical computation. Yet, at several places in the text, the author feels it necessary to
almost apologize for addressing "theoretical" questions. The book itself is a counterexample to the notion that practical and theoretical matters lead separate lives.

In conclusion, Strang has written an excellent text which gives valuable insights into both theoretical and practical questions. For those with an applied bent it can serve as the basis for an excellent course, and, it should not be overlooked as a text for mathematics majors. The innovative treatments in Strang will undoubtedly be "adopted" by many other authors in the future. Can there be any better compliment?

NICHOLAS J. ROSE
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This is a second edition of Cole's well-known book, revised and updated in certain respects. The team of authors is a felicitous one and highly qualified to expound the two particular methods—matching and multiscale procedures—which the book aims to teach. A signal improvement over the first edition appears to this reviewer to lie in a clarification of the distinction, and relation, between the two methods. Much of the chapter on the two-scale procedure, moreover, is substantially new and offers the most serious and penetrating treatment of this method which I have found in a book to date. Certainly, any student of the multiscale procedure should read Chapter 3 with care.

On the other hand, there are severe limitations on what even so outstanding a team of authors can achieve with a second edition in a subject under intensive development. The attempt at updating could not possibly transcend the narrowest of limits and the reader may find the references at the ends of Sections useful for distinguishing those in which any updating was attempted at all. Even such indications must be interpreted with care, however; for instance, the one Section entitled "Rigorous Results" quotes but a single theorem out of a large body of mathematical theory built up over the last two decades and refers to one isolated, recent paper out of hundreds. Updating was here clearly impossible within the framework of a second edition, but this might have been admitted more frankly. Certainly, any serious student of "limit-expansions" or "matching" should read Eckhaus' Asymptotic Analysis of Singular Perturbations (North-Holland, Amsterdam, 1979).

The pedagogical principle of the book remains the exposition of procedural methods at the hand of examples. The great value of this approach for graduate courses is obvious, but naturally, there is a price that needs to be recognized also. The principle leads more readily to use of an example to illustrate a method, than of the method to illustrate the subject from which the example is drawn. To blend both objectives is not humanly possible on the scale of this book, and it has been achieved in only a relatively few sections. In most, the method is served, but not the subject—to a degree making this one of the more serious reasons why the book is still not really suitable as a text. Rather, it is a valuable source of material for an instructor, particularly on the multiscale procedure, provided the material be selected carefully.

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This is a brief well written valuable book which will be of service to both students and researchers. In 121 pages Roth rings the changes on seventeen different properties suggested as axioms by various individuals in the formalization of bargaining models.