Substitution in Double Integrals. Jacobian.

[1] Let $D$ be the parallelogram bounded by lines

$$3x + y = 1, \quad 3x + y = 3, \quad x - y = -1, \quad x - y = 4.$$ 

Evaluate $\iint_D x \, dA$.

[2] Let $D$ be the quadrilateral in the $xy$ plane defined by

$$1 \leq 4x + y \leq 2, \quad x \geq 0, \quad y \geq 0.$$ 

Evaluate $\iint_D e^{(x+y)/(4x+y)} \, dA$.

[3] Find the area enclosed by the curve $x^{2/3} + y^{2/3} = 1$ in the first quadrant.

(See next page for the answers)
Answers:

[1] $35/16$
(Hint: Substitute $u = 3x + y, v = x - y$, or, equivalently, $x = (u + v)/4, y = (u - 3v)/4$. The parallelogram $D$ in the $xy$ plane corresponds to a rectangle in the $uv$-plane bounded by $u = 1, u = 3, v = -1, v = 4$.)

[2] $(e - e^{1/4})/2$
(Hint: Substitute $u = 4x + y, v = x$, or, equivalently, $x = v, y = u - 4v$. The quadrilateral $D$ in the $xy$ plane corresponds to a quadrilateral $\Omega$ in the $uv$-plane given by $1 \leq u \leq 2, v \geq 0, u - 4v \geq 0$.)

[3] $3\pi/32$
(Hint: Substitute $u = x^{1/3}, v = y^{1/3}$, or, equivalently, $x = u^3, y = v^3$. The required region $D$ in the $xy$ plane corresponds to $\Omega$, which is one fourth of the unit disk on the $uv$-plane.)