Substitution in Triple Integrals. Cylindrical and Spherical Coordinates.

[1] Let $V$ be the solid region that is inside the cylinder $x^2 + y^2 = 4$, below the paraboloid $z = x^2 + y^2 - 2$, and above the $xy$-plane. Find the centroid of the solid.

[2] Let $V$ be the upper half of the unit ball $x^2 + y^2 + z^2 \leq 1$, $z \geq 0$. Evaluate $\iiint_V z e^{-x^2-y^2-z^2} \, dV$.

[3] Let $V$ be the ellipsoid bounded by

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2} = 1.$$

Evaluate $\iiint_V |x| \, dV$.

[4] Let $V$ be the solid region defined by

$$x^2 + 4(1 + x)^2 y^2 + z^2 < 4, \quad x > 0, y > 0, z > 0.$$

Evaluate $\iiint_V (1 + x)^2 \, dV$.

(See next page for the answers)
Answers:

[1] $x = y = 0, z = 2/3$
   (Hint: Use cylindrical coordinates.)

[2] $\pi/2 - \pi/e$
   (Hint: Use spherical coordinates.)

[3] $\frac{1}{2} \pi a^2 bc$
   (Hint: First substitute $x = au, y = bv, z = cw$. Then consider spherical coordinates for $(u, v, w)$.)

[4] $7\pi/6$
   (Hint: Substitute $u = x, v = 2(1 + x)y, w = z$, or, equivalently, $x = u, y = \frac{v}{2(1+u)}, z = w$. The solid $V$ corresponds to a part of a ball in the $uvw$-space.)