Exercises on Isolated Singularities

Find all singular points of the given function $f(z)$ on the complex plane. For each singular point, answer the following questions:

- Is it an isolated singularity?

- Is it a removable singularity, a pole, or an essential singularity? If $z = z_0$ is a removable singularity, give the function value $f(z_0)$ to make $f(z)$ complex differentiable at $z_0$. If $z_0$ is a pole, determine its order.

(a) $\frac{z + 1}{z^2(z^2 - 1)^3}$   (b) $\frac{z}{\cos z - 1}$   (c) $\frac{z}{\sin z}$   (d) $e^{-3/z^2}$   (e) $z^7 \cos \left(\frac{1}{z}\right)$

(f) $\log z = \ln r + i\theta$ for $z = re^{i\theta}$ with $r > 0, -\pi < \theta < \pi$

(g) $\sum_{n=1}^{\infty} \frac{1}{(z^2 + n^2)^{3n}}$