Electric Circuits

& Diff Eq's

- Faraday's Law

L : inductance
i : electric current

$$(\text{voltage drop across the inductor}) = L\frac{di}{dt}$$

- Ohm's Law

R : resistance
i : electric current

$$(\text{voltage drop across the resistor}) = Ri$$

- Coulomb's Law

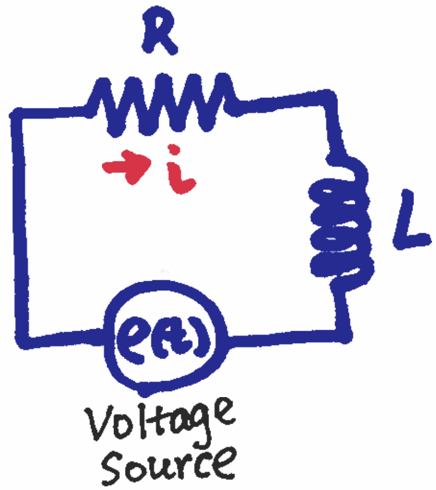
C : capacitance
Q : electric charge on capacitor

- $(\text{voltage drop across the capacitor}) = \dfrac{Q}{C}$
- electric current $i = \dfrac{dQ}{dt}$

- Kirchhoff's voltage Law

$$(\text{the sum of voltage drops in any loop}) = 0$$

RL Circuit

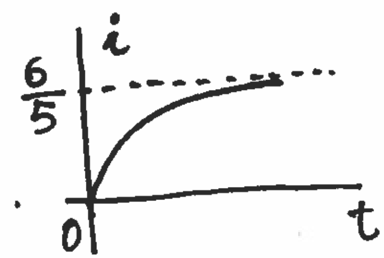


- **Diff Eq:** $L\frac{di}{dt} + Ri - e(t) = 0$ [1st order lin. diff. eq solved by integrating factor]
- **Gen. Sol:** $i(t) = e^{-\frac{R}{L}t}\int \frac{e(t)}{L} e^{\frac{R}{L}t}\,dt + Ke^{-\frac{R}{L}t}$ where K is a free parameter
- **Case $e(t) = E_0$** (constant voltage source)

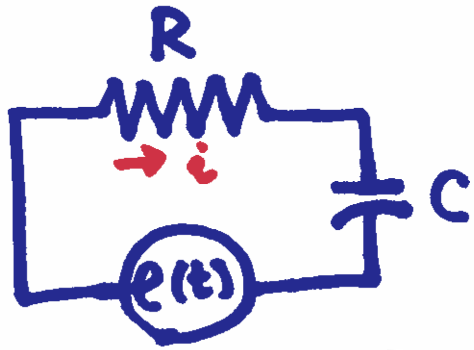
$$i(t) = \frac{E_0}{R} + Ke^{-\frac{R}{L}t}$$

- **Example** $L = \frac{1}{2}$ henry, $R = 10$ ohm, $e(t) = E_0 = 12$ volt, $i(0) = 0$.

$$\Rightarrow i(t) = \frac{12}{10} - \frac{12}{10} e^{-\frac{10}{1/2}t} = \frac{6}{5} - \frac{6}{5} e^{-20t} \text{ (ampere).}$$



RC Circuit



$$Ri + \frac{Q}{C} - e(t) = 0$$

Dif Eq for Q (Charge on the capacitor)

$$R\frac{dQ}{dt} + \frac{1}{C}Q - e(t) = 0 \qquad (*)$$

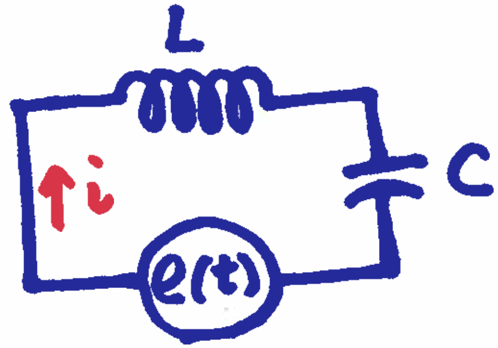
Dif Eq for i (current) $\Leftarrow$ Apply $\frac{d}{dt}$ in (*).

$$R\frac{di}{dt} + \frac{1}{C}i - e'(t) = 0$$

Dif Eq for v (voltage drop across the capacitor) $\Leftarrow$ Q replaced by Cv in (*).

$$RC\frac{dv}{dt} + v - e(t) = 0$$

LC Circuit



$$\begin{cases} L\frac{di}{dt} + \frac{Q}{C} - e(t) = 0 \\ i = \frac{dQ}{dt} \end{cases}$$

A System of Diff Eq's for Q and i

$$\begin{cases} \frac{dQ}{dt} = i \\ \frac{di}{dt} = -\frac{1}{CL}Q + \frac{e(t)}{L} \end{cases}$$

$$\frac{d}{dt}\begin{bmatrix} Q \\ i \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ -\frac{1}{CL} & 0 \end{bmatrix}\begin{bmatrix} Q \\ i \end{bmatrix} + \begin{bmatrix} 0 \\ \frac{e(t)}{L} \end{bmatrix}$$

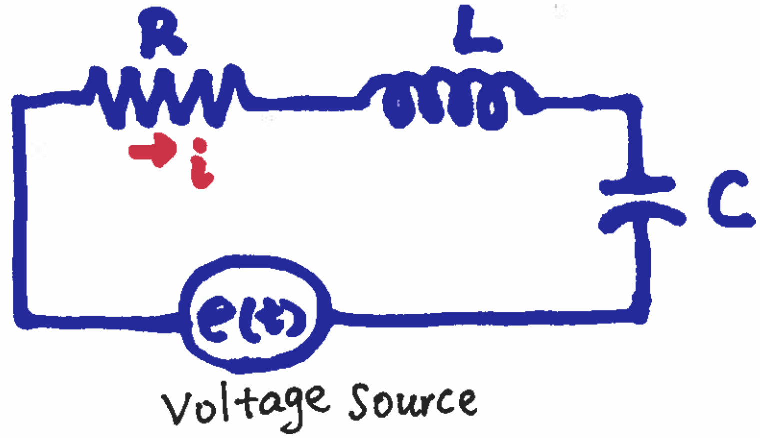
A 2nd order diff eq for Q

$$L\frac{d^2Q}{dt^2} + \frac{1}{C}Q - e(t) = 0 .$$

A System of Diff Eqs for v and i:

$$\begin{cases} \frac{dv}{dt} = \frac{1}{C}i \\ \frac{di}{dt} = -\frac{1}{L}v + \frac{e(t)}{L} \end{cases}$$

RLC Circuit



$$(*) \quad Ri + L\frac{di}{dt} + \frac{Q}{C} - e(t) = 0$$

$$i = \frac{dQ}{dt}$$

$$v = \frac{Q}{C}$$

Diff Eq. for Q (electric charge on the capacitor)

$$L\frac{d^2Q}{dt^2} + R\frac{dQ}{dt} + \frac{Q}{C} = e(t)$$

Diff Eq. for i (current)

$$L\frac{d^2i}{dt^2} + R\frac{di}{dt} + \frac{1}{C}i = e'(t)$$

Diff Eq. for v (voltage drop across the capacitor)

$$LC\frac{d^2v}{dt^2} + RC\frac{dv}{dt} + v = e(t)$$