

Ans. Key

Math 2551 A1-3 Exercise 10

Section:

Name:

Student Number:

Let

$$f(x, y) = \begin{cases} \frac{x^2 y^2}{x^2 + y^2}, & (x, y) \neq (0, 0), \\ 1, & (x, y) = (0, 0). \end{cases}$$

Mark "true" or "false" for each of the following statements.

True (1) $\lim_{(x,y) \rightarrow (0,0)} f(x, y)$ exists;

False (2) $f(x, y)$ is continuous at $(0, 0)$.

$$(1) \quad \text{When } (x, y) \neq (0, 0), \quad 0 \leq \left| \frac{x^2 y^2}{x^2 + y^2} \right| \leq \left| \frac{\frac{1}{2}(x^2 + y^2) \cdot xy}{x^2 + y^2} \right| = \frac{1}{2} |xy|$$

\downarrow \downarrow as $(x, y) \rightarrow (0, 0)$

$\therefore \lim_{(x,y) \rightarrow (0,0)} f(x, y) = 0$ by the Pinching thm.

$$(2) \quad f(0, 0) = 1 \neq \lim_{(x,y) \rightarrow (0,0)} f(x, y) = 0$$

Therefore $f(x, y)$ is NOT cont. at $(0, 0)$