

Ans. Key

Math 2551 A1-3 Exercise 12

Section:

Name:

Student Number:

Let $f(x, y)$ be a function having continuous partial derivatives up to 2nd order. Let $x = r \cos \theta$, $y = r \sin \theta$. Mark "true" or "false" for each of the following statements.

True (1) $\frac{\partial f}{\partial r} = \frac{\partial f}{\partial x} \cos \theta + \frac{\partial f}{\partial y} \sin \theta$;

$$\frac{\partial f}{\partial r} = \frac{\partial f}{\partial x} \frac{\partial x}{\partial r} + \frac{\partial f}{\partial y} \frac{\partial y}{\partial r} = \frac{\partial f}{\partial x} \cos \theta + \frac{\partial f}{\partial y} \sin \theta$$

False (2) $\frac{\partial^2 f}{\partial \theta \partial r} = \frac{\partial^2 f}{\partial x^2} \cos \theta - \frac{\partial f}{\partial x} \sin \theta + \frac{\partial^2 f}{\partial y^2} \sin \theta + \frac{\partial f}{\partial y} \cos \theta$;

True (3) $\frac{\partial^2 f}{\partial \theta \partial r} = \frac{\partial}{\partial \theta} \left(\frac{\partial f}{\partial x} \right) \cos \theta - \frac{\partial f}{\partial x} \sin \theta + \frac{\partial}{\partial \theta} \left(\frac{\partial f}{\partial y} \right) \sin \theta + \frac{\partial f}{\partial y} \cos \theta$;

True (4) $\frac{\partial}{\partial \theta} \left(\frac{\partial f}{\partial x} \right) = -\frac{\partial f^2}{\partial x^2} r \sin \theta + \frac{\partial f^2}{\partial y \partial x} r \cos \theta$.

$$\begin{aligned} \frac{\partial}{\partial \theta} \left(\frac{\partial f}{\partial x} \right) &= \frac{\partial}{\partial x} \left(\frac{\partial f}{\partial x} \right) \cdot \frac{\partial x}{\partial \theta} + \frac{\partial}{\partial y} \left(\frac{\partial f}{\partial x} \right) \cdot \frac{\partial y}{\partial \theta} \\ &= \frac{\partial^2 f}{\partial x^2} (-r \sin \theta) + \frac{\partial^2 f}{\partial y \partial x} (r \cos \theta) \end{aligned}$$

$$\begin{aligned} \frac{\partial}{\partial \theta} \left(\frac{\partial f}{\partial y} \right) &= \frac{\partial}{\partial x} \left(\frac{\partial f}{\partial y} \right) \cdot \frac{\partial x}{\partial \theta} + \frac{\partial}{\partial y} \left(\frac{\partial f}{\partial y} \right) \cdot \frac{\partial y}{\partial \theta} \\ &= \frac{\partial^2 f}{\partial x \partial y} (-r \sin \theta) + \frac{\partial^2 f}{\partial y^2} (r \cos \theta) \end{aligned}$$