

Math 2551 Exercise 18

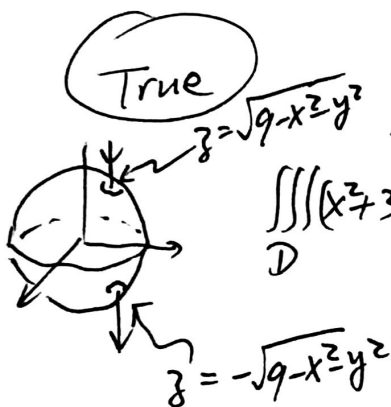
Section:

Name:

Student ID:

Let $D = \{(x, y, z) : x^2 + y^2 + z^2 \leq 9\}$ and $R = \{(x, y) : x^2 + y^2 \leq 9\}$. Mark true or false for each of the following statements.

(1)



True

$$\iiint_D (x^2 + 3) dx dy dz = \int_0^{2\pi} \int_0^3 \left\{ \int_{-\sqrt{9-r^2}}^{\sqrt{9-r^2}} (r^2 \cos^2 \theta + 3) dz \right\} r dr d\theta.$$

$$\iiint_D (x^2 + 3) dv = \iint_R \left(\int_{-\sqrt{9-x^2-y^2}}^{\sqrt{9-x^2-y^2}} (x^2 + 3) dz \right) dx dy$$

Polar coord change

$$\int_0^{2\pi} \int_0^3 \left\{ \int_{-\sqrt{9-r^2}}^{\sqrt{9-r^2}} (r^2 \cos^2 \theta + 3) dz \right\} r dr d\theta$$

False

(2) The map $x = -u$, $y = v$ and $z = w$ transforms the solid G in uvw -coordinate into solid D , then

$$\iiint_D (x^2 + 3) dx dy dz = - \iiint_G (u^2 + 3) du dv dw$$

$$\frac{\partial(x, y, z)}{\partial(u, v, w)} = \det \begin{pmatrix} -1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} = -1, \quad \left| \frac{\partial(x, y, z)}{\partial(u, v, w)} \right| = 1$$

$$\begin{aligned} \therefore \iiint_D (x^2 + 3) dv &= \iiint_G (u^2 + 3) \left| \frac{\partial(x, y, z)}{\partial(u, v, w)} \right| du dv dw \\ &= \iiint_G (u^2 + 3) du dv dw \end{aligned}$$