Ans. key

Math 2551 A1-3 Exercise 24

Section:
Name:
Student ID:

Let R be the region enclosed by the ellipse C : $\mathbf{r}(t) = 2\cos t\mathbf{i} + \sin t\mathbf{j}$, $t \in [0, 2\pi]$, and $\mathbf{n}$ be its outward pointing unit normal vector. Let $\mathbf{F}(x, y) = f(x, y)\mathbf{i} + g(x, y)\mathbf{j}$, where f and g are continuous functions.

Mark true or false for each of the following statements.

False (1) $\int_C f\,ds = \int_0^{2\pi} f(\cos t, \sin t)\frac{ds}{dt}dt$.

$\llcorner \int_0^{2\pi} f(2\cos t, \sin t)\,\frac{ds}{dt}\,dt$

True (2) $\int_C \mathbf{F}\cdot d\mathbf{r} = \int_0^{2\pi} \mathbf{F}\cdot\frac{d\mathbf{r}}{dt}dt$.

True (3) $\int_C f\,dx = \int_C (f\mathbf{i} + 0\mathbf{j})\cdot d\mathbf{r}$.

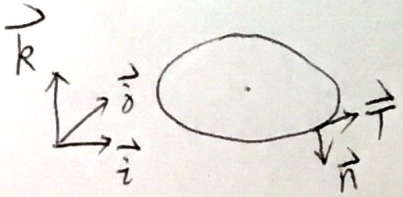
since $d\vec{r} = dx\,\vec{i} + dy\,\vec{j}$

$\therefore f\,dx = (f\vec{i} + 0\vec{j})\cdot d\vec{r}$

False (4) $\int_C \mathbf{F}\cdot\mathbf{n}\,ds = \int_C f\,dx + g\,dy$.

Note that $\vec{n} = \vec{T}\times\vec{k} = \frac{d\vec{r}}{ds}\times\vec{k} = \frac{dx}{ds}\vec{i}\times\vec{k} + \frac{dy}{ds}\vec{j}\times\vec{k}$

$= -\frac{dx}{ds}\vec{j} + \frac{dy}{ds}\vec{i}$



$\therefore \int_C \vec{F}\cdot\vec{n}\,ds = \int_C \left(f\frac{dy}{ds} - g\frac{dx}{ds}\right)ds = \int_C -g\,dx + f\,dy$