Ans. key

Math 2551 A1-3 Exercise 25

Section:
Name:
Student Number:

Let $\mathbf{r}(t) = f_1(t)\mathbf{i} + f_2(t)\mathbf{j} + f_3(t)\mathbf{k}$, $t \in [1, 2]$ parametrize a differentiable curve C in R^3. Then $\int_C y\,dy + x^2 dx$ is equal to (mark "true" or "false" for each answer.)

True (A) $\int_C [x^2\mathbf{i} + y\mathbf{j}] \cdot d\mathbf{r}$. $\int_C y\,dy + x^2 dx = \int_C (x^2\vec{i} + y\vec{j} + 0\vec{k}) \cdot (dx\vec{i} + dy\vec{j} + dz\vec{k})$

True (B) $\int_1^2 [f_1^2(t) f_1'(t) + f_2(t) f_2'(t)]dt$.

False (C) $(\frac{1}{3}x^3 + \frac{1}{2}y^2)|_{(x,y)=(1,1)}^{(x,y)=(2,2)}$.

True (D) $\frac{1}{3}(f_1^3(2) - f_1^3(1)) + \frac{1}{2}(f_2^2(2) - f_2^2(1))$.

(B) $\int_C y\,dy + x^2 dx = \int_1^2 f_2(t)\, d f_2(t) + [f_1(t)]^2 d f_1(t)$

$= \int_1^2 f_2(t) f_2'(t) dt + f_1^2(t) f_1'(t)\, dt = \int_1^2 [\,\cdot\,\cdot\,]\, dt$

(C) Even though $\nabla(\frac{1}{3}x^3 + \frac{1}{2}y^2) = x^2\vec{i} + y\vec{j}$, the starting and ending pts of curve C are NOT $(1,1)$ or $(2,2)$, they are $(f_1(1), f_2(1), f_3(1))$ and $(f_1(2), f_2(2), f_3(2))$ respectively.

(D) $\int_C y\,dy + x^2 dx = \left(\frac{1}{3}x^3 + \frac{1}{2}y^2\right)\Big|_{(f_1(1), f_2(1), f_3(1))}^{(f_1(2), f_2(2), f_3(2))}$