Ans. key

Math 2551 A1-3 Exercise 5

Section:
Name:
Student Number:

Let a curve be parametrized by $\mathbf{r}(t) = \sin t\mathbf{i} + \mathbf{j} + e^t\mathbf{k}$, $t \geq 0$, and $s(t)$ be the arc length from $\mathbf{r}(0)$ to $\mathbf{r}(t)$. Mark "True" or "False" for each of the following statements.

False (1) $|\frac{d\mathbf{r}}{dt}| = 1$ for $t > 0$;

True (2) $|\frac{d\mathbf{r}}{ds}| = 1$ for $s > 0$;

True (3) $\frac{d\mathbf{r}}{ds}$ is parallel to $\frac{d\mathbf{r}}{dt}$ at any point of the curve.

True (4) $s(t)$ is an increasing function for $t \geq 0$.

In (1), $\frac{d\vec{r}}{dt}$ doesn't have unit length in general. In fact, $\left|\frac{d\vec{r}}{dt}\right| = \sqrt{\cos^2 t + e^{2t}} \neq 1$ for $t > 0$.

In (2), $\frac{d\vec{r}}{ds} = \frac{d\vec{r}}{dt} \Big/ \frac{ds}{dt} = \frac{d\vec{r}}{dt} \Big/ \left|\frac{d\vec{r}}{dt}\right|$, $\therefore \left|\frac{d\vec{r}}{ds}\right| = 1$

In (3), $\frac{d\vec{r}}{ds} = \frac{d\vec{r}}{dt} \cdot \frac{dt}{ds}$, therefore $\frac{d\vec{r}}{dt} \parallel \frac{d\vec{r}}{ds}$

In (4), $s'(t) = \left|\frac{d\vec{r}}{dt}\right| > 0$ (see (1)), thus $s(t)$ is increasing.