Ans. key

Math 2551 A1-3 Exercise 6

Section:
Name:
Student Number:

Let $\mathbf{r}(t) = \cos 3t\mathbf{i} + \sin 3t\mathbf{j} + e^t\mathbf{k}$, $s(t)$ be the arc length parameter from $\mathbf{r}(0)$ to $\mathbf{r}(t)$. Mark "true" or "false" for each of the following statements.

True (1) $\mathbf{r}'(t) = s'(t)\mathbf{T}$;

True (2) T will change direction if we change parameter t to $\mu = -t$;

True (3) $d\mathbf{T}/ds = k\mathbf{N}$;

False (4) $\mathbf{N}$ will change direction if we change parameter t to $\mu = -t$.

(1). $\vec{T} = \frac{d\vec{r}}{dt} \Big/ \left|\frac{d\vec{r}}{dt}\right|$, $\therefore$ $\vec{T}$ has the same dir. as $\frac{d\vec{r}}{dt}$. Also $|\vec{r}'(t)| = s'(t)$, thus $\vec{r}'(t) = s'(t)\vec{T}$ because they have the same dir and length.

(2) $\frac{d\vec{r}}{d\mu} = \frac{d\vec{r}}{dt} \cdot \frac{dt}{d\mu} = -\frac{d\vec{r}}{dt}$, thus the direction changes.

(3) $\left|\frac{d\vec{T}}{ds}\right| = k$, $\frac{d\vec{T}}{ds}$ has the same dir as $\vec{N}$.

(4) $\vec{N}$ is independent of the parameter.