Ans. key

Math 2551 A1-3 Exercise 9

Section:
Name:
Student ID:

Let $f(x, y)$ and $g(x, y)$ be functions defined on the whole xy-plane. Mark true or false for each of the following statements.

True (1) If $\lim_{(x,y)\to(1,2)} f(x, y)$ exists and $\lim_{(x,y)\to(1,2)} g(x, y)$ doesn't exist, then $\lim_{(x,y)\to(1,2)}[f(x, y)+g(x, y)]$ doesn't exist; In fact, if $\lim(f+g)$ exists, then $\lim(f+g)-f$ exists because both $f+g$ & f have limits, thus $\lim g$ exists, $((f+g)-f=g)$, contradictory to the assumption.

False (2) If $\lim_{(x,y)\to(1,2)} f(x, y)$ exists and $\lim_{(x,y)\to(1,2)} g(x, y)$ doesn't exist, then $\lim_{(x,y)\to(1,2)}[f(x, y)g(x, y)]$ doesn't exist; For example, $f\equiv 0$, $g=\cdots$, then $fg\equiv 0$, and $\lim fg = 0$ exists.

False (3) If $\lim_{(x,y)\to(1,2)} f(x, y)$ exists and $\lim_{(x,y)\to(1,2)} g(x, y)$ doesn't exist, then $\lim_{(x,y)\to(1,2)}[f(x, y)/g(x, y)]$ doesn't exist; For example, $f\equiv 0$, g is such that $\frac{1}{g}$ is well defined but $\lim g$ doesn't exist (find such function!), then $\frac{f}{g}\equiv 0$ thus $\lim \frac{f}{g}$ exists.

False (4) If $\lim_{(x,y)\to(1,2)} f(x, y)$ exists and $\lim_{(x,y)\to(1,2)} g(x, y)$ exists, then $\lim_{(x,y)\to(1,2)}[f(x, y)/g(x, y)]$ exists.

For example, $f\equiv 1$, $g\equiv 0$, but $\frac{f}{g}$ isn't defined, let alone the limit.

In summary, to say a statement is true, prove it!
To say a statement is not true, find an example.