Math 2551 A1-3 Final (for Practice)

Section:
Name:
Student ID:

(1) Solve the initial value problem for $\mathbf{r}$ as a vector function of t.

$$\frac{d^2\mathbf{r}}{dt^2} = -(\mathbf{i} + \mathbf{j} + \mathbf{k}),$$
$$\mathbf{r}(0) = 10\mathbf{i} + 10\mathbf{j} + 10\mathbf{k} \text{ and } \frac{d\mathbf{r}}{dt}|_{t=0} = \mathbf{0}.$$

(2) Find an equation for the circle of curvature of the circle $\mathbf{r}(t) = 2\ln t\mathbf{i} - (t + 1/t)\mathbf{j}$, $e^{-2} \leq t \leq e^2$, at the point $(0, -2)$, where $t = 1$.

(3) Let

$$f(x,y) = \begin{cases} \frac{xy^2}{x^2+y^4}, & (x,y) \neq (0,0) \\ 0, & (x,y) = (0,0). \end{cases}$$

Show that $f_x(0,0)$ and $f_y(0,0)$ exist, but f is not differentiable at $(0,0)$.

(4) Let $w = f(u)+g(v)$, where $u = x+iy$, $v = x-iy$ and $i = \sqrt{-1}$. Does w satisfy the Laplace equation $w_{xx}+w_{yy} = 0$ if all necessary functions are differentiable? Justify your answer.

(5) Find the absolute maxima and minima of the function on the given domain. $f(x, y) = 4x - 8xy + 2y + 1$ on the triangular plate bounded by the lines $x = 0$, $y = 0$, $x + y = 1$ in the first quadrant.

(6) Find the volume of the region that lies inside the sphere $x^2 + y^2 + z^2 = 2$ and outside the cylinder $x^2 + y^2 = 1$.

(7) Find the counterclockwise circulation and outward flux for the field $\mathbf{F}$ and curve C.

$$\mathbf{F} = (y^2 - x^2)\mathbf{i} + (x^2 + y^2)\mathbf{j}.$$

C: The triangle bounded by $y = 0$, $x = 3$, and $y = x$.

(8) Find the flux $\int\int_S \mathbf{F} \cdot \mathbf{n} d\sigma$ across the surface in the specified direction. Here $\mathbf{F} = -x\mathbf{i} - y\mathbf{j} + z^2\mathbf{k}$ outward (normal away from the z-axis) through the portion of the corn $z = \sqrt{x^2 + y^2}$ between the planes $z = 1$ and $z = 2$.