$$\frac{d^2x}{dt^2} + \varepsilon (x^2-1) \frac{dx}{dt} + x = 0.$$ 

$$\Rightarrow \frac{dx}{dt} = y,$$
$$\frac{d^2x}{dt^2} = \frac{dy}{dt} = \varepsilon (x^2-1) y - y .$$

$$\Rightarrow \frac{dx}{dt} = y,$$
$$\frac{dy}{dt} = -\varepsilon (x^2-1) y - y .$$

from $\frac{dx}{dt} = 0, \frac{dy}{dt} = 0$.

we know that $(0,0)$ is the only equilibrium point.

The linearized system of a given system at $(0,0)$ is

$$\begin{bmatrix}
\frac{dx}{dt} \\
\frac{dy}{dt}
\end{bmatrix} = A \cdot \begin{bmatrix}
x \\
y
\end{bmatrix}, \quad A = Df(0,0) = \begin{bmatrix}
0 & 1 \\
-1 & -\varepsilon
\end{bmatrix} .$$

$$\det (A - \lambda I) = \begin{vmatrix}
0 - \lambda & 1 \\
-1 & -\varepsilon - \lambda
\end{vmatrix} = \lambda (\lambda - \varepsilon) + 1 = \lambda^2 - \varepsilon \lambda + 1$$

$$\therefore \text{eigenvalues of } A , \quad \lambda_1, \lambda_2 = \frac{-\varepsilon \pm \sqrt{\varepsilon^2 - 4}}{2} ,$$

(1) \(\varepsilon^2 - 4 < 0 \Rightarrow \lambda_1, \lambda_2 \text{ are conjugate complex numbers.}

\( \varepsilon > 0 \Rightarrow 0 < \varepsilon < 2 \)

$(0,0)$ is unstable spiral.
(ii) $\varepsilon < 0$

$-2 \leq \varepsilon < 0$

$(0, 0)$ is a stable spiral (focus).

(iii) $\varepsilon^2 - 4 > 0$

$\varepsilon > 2$

$\lambda_1, \lambda_2 > 0$

$(0, 0)$ is an unstable node

$\varepsilon < -2$

$\lambda_1, \lambda_2 < 0$

$(0, 0)$ is a stable node
\[ \theta_1 = \arg(z-1), \quad \theta_2 = \arg(z-2). \]

From addition, \[ \arg(z-1) - \arg(z-2) = \frac{\pi}{3}. \]

We know that \[ \overrightarrow{OZ} \perp \overrightarrow{ZA}. \]

Thus, we can use Pythagorean theorem:
\[ \sqrt{x^2+y^2} + \sqrt{(x-1)^2+y^2} = 2. \]

\[ x^2 + y^2 - 2x + 1 + y^2 = 4 \]
\[ 2x^2 - 4x + 2y^2 = 0 \]
\[ x^2 - 2x + 1 + y^2 = 0 \]
\[ (x-1)^2 + y^2 = 1. \]

Since \[ 0 \leq \arg z \leq \frac{\pi}{2} \], the trajectory of \( z \) is a semicircle arc of radius 1 in the upper half of the \( z \)-plane, with center at \((1,0)\).
(- R equation : \( \frac{du}{dx} = \frac{du}{dy} \), \( \frac{du}{dy} = -\frac{du}{dx} \).) \quad - (1)

\[ x = r \cos \theta, \quad y = r \sin \theta. \]

\[ \frac{du}{dx} = \frac{du}{dr} \cdot \frac{dr}{dx} + \frac{du}{d\theta} \cdot \frac{d\theta}{dx} = \frac{1}{\cos \theta} \cdot \frac{du}{dr} - \frac{1}{r \sin \theta} \frac{du}{d\theta}. \]

\[ \frac{dv}{dy} = \frac{dv}{dr} \cdot \frac{dr}{dy} + \frac{dv}{d\theta} \cdot \frac{d\theta}{dy} = \frac{1}{\sin \theta} \cdot \frac{dv}{dr} + \frac{1}{r \cos \theta} \frac{dv}{d\theta}. \]

\[ \frac{du}{dy} = \frac{du}{dr} \cdot \frac{dr}{dy} + \frac{du}{d\theta} \cdot \frac{d\theta}{dy} = \frac{1}{\tan \theta} \cdot \frac{du}{dr} + \frac{1}{r \sin \theta} \frac{du}{d\theta}. \]

\[ \frac{dv}{dx} = \frac{dv}{dr} \cdot \frac{dr}{dx} + \frac{dv}{d\theta} \cdot \frac{d\theta}{dx} = \frac{1}{\tan \theta} \cdot \frac{dv}{dr} - \frac{1}{r \sin \theta} \frac{dv}{d\theta}. \]

Plug in (2) to (1).

\[ \frac{1}{r \cos \theta} \frac{du}{dx} - \frac{1}{r \sin \theta} \frac{du}{d\theta} = \frac{1}{\sin \theta} \frac{du}{dr} + \frac{1}{r \cos \theta} \frac{dv}{d\theta}. \quad - (3) \]

\[ \frac{1}{\sin \theta} \frac{du}{dx} + \frac{1}{r \cos \theta} \frac{du}{d\theta} = - \frac{1}{r \cos \theta} \frac{dv}{dr} + \frac{1}{r \sin \theta} \frac{dv}{d\theta}. \quad - (4) \]

\[ (3) \times \frac{1}{r \cos \theta} + (4) \times \frac{1}{\sin \theta} \Rightarrow \quad \frac{du}{dr} = \frac{1}{r} \frac{dv}{d\theta}. \quad (\text{R equation}) \]

\[ (3) \times \frac{1}{\sin \theta} - (4) \times \frac{1}{r \cos \theta} \Rightarrow - \frac{1}{r} \frac{du}{d\theta} = \frac{dv}{dr} \] in polar coordinate.