Math 1502   Quiz1   Summer 2007

Solutions for section B1, B2

Problem 1

Find the least positive integer n so that the $n-th$ degree Taylor Polynomial $P_n(x)$ in x approximates $cos 0.3$ to within 0.01.

Solution: The $2n-th$ reminder of $f(x) = cosx$ is

$$R_{2n}(x) = \frac{f^{2n+2}(c)}{(2n+2)!}x^{2n+2}$$

We want $|R_{2n}(0.3)| < 0.01$.

Since $|(cosx)^{(n)}| \leq 1$ for all n and x, we only need to find the least n such that $\frac{0.3^{2n+2}}{(2n+2)!} < 0.01$.

It's easy to see $n = 1$ is the least integer, and $2n = 2$. So the $2nd$ degree Taylor Polynomial of $cosx$ approximates $cos 0.3$ to within 0.01.

Problem 2

Find $lim_{x\to+\infty}(1+e^x)^{1/x}$.

Solution: This is the ∞^0 form, so we take the natural log first.

$$\begin{aligned} lim_{x\to+\infty} ln(1+e^x)^{1/x} &= lim_{x\to+\infty}\frac{1}{x}ln(1+e^x) \\ &= lim_{x\to+\infty}\frac{ln(1+e^x)}{x} \\ &= lim_{x\to+\infty}\frac{e^x}{1+e^x} \qquad (applying\ \ L'Hospital's\ \ Rule) \\ &= lim_{x\to+\infty}\frac{e^x}{e^x} \qquad (applying\ \ L'Hospital's\ \ Rule\ \ again) \\ &= 1 \end{aligned}$$

So $lim_{x\to+\infty}(1+e^x)^{1/x} = e^1 = e$.