Problem 1
Find the general solution of \( y' - 2y = e^x \).
Solution: This is a 1st order linear equation with standard form \( y' + P(x)y = Q(x) \).
Here \( P(x) = -2, Q(x) = e^x \), so the integrating factor \( H(x) = \int P(x)dx = \int -2dx = -2x \).
The general solution is
\[
\begin{align*}
y(x) &= e^{-H(x)}(\int e^{H(x)}Q(x)dx + C) \\
&= e^{2x}(\int e^{-2x}e^x dx + C) \\
&= e^{2x}(-e^x + C)
\end{align*}
\]

Problem 2
Solve the initial value problem: \( x^2y' = y - xy, \ y(1) = 1 \).
Solution: This is a separable equation.
\[
\frac{dy}{dx} = y' = \frac{y(1-x)}{x^2}
\]
so
\[
\frac{dy}{y} = \frac{(1-x)}{x^2}dx
\]
Take the integral on both sides:
\[
\begin{align*}
\ln(y) &= \int \frac{(1-x)}{x^2}dx + C \\
&= \int \frac{1}{x^2} - \frac{1}{x}dx + C \\
&= -x^{-1} - \ln(x) + C
\end{align*}
\]
Plug in the initial value \( y(1) = 1 \) to calculate the constant \( C \):
\[
\ln(1) = -1 - \ln(1) + C \implies C = 1.
\]
Thus \( \ln(y) = -x^{-1} - \ln(x) + 1 \).