Math6014, Take Home Exam, Due: 2pm December 5, 2005

- I commit to uphold the ideals of honor and integrity by refusing to betray the trust bestowed upon me as a member of Georgia Tech community.

  SIGNATURE:

- Please do all three problems.

1. For each $i \in \{1,2,3\}$, let G_i denote a connected graph and let x_i, y_i, z_i be three distinct vertices of G_i. Let G be the graph obtained from the disjoint union of G_1, G_2, G_3 by adding the following edges: $x_1x_2, x_2x_3, x_3x_1, y_1y_2, y_2y_3, y_3y_1$. Assume G is 4-colorable. Show that G has a 4-coloring in which z_1, z_2, z_3 receive pairwise distinct colors.

2. Let G be a graph with a 4-flow. Show that there is a collection $\mathcal{C}$ of cycles in G such that each edge of G occurs in exactly two members of $\mathcal{C}$.

3. Prove that $t_{r-1}(n) \leq \frac{1}{2}n^2\frac{r-2}{r-1}$, with equality iff $r-1$ divides n.

NOTE: There will be a 2 hour lecture on Friday, December 2. There will be no lectures on December 7 and 9.