Math6014, Take Home Exam, Due: 2pm December 5, 2005

• I commit to uphold the ideals of honor and integrity by refusing to betray the trust bestowed upon me as a member of Georgia Tech community.

**Signature:**

• Please do all three problems.

1. For each \( i \in \{1, 2, 3\} \), let \( G_i \) denote a connected graph and let \( x_i, y_i, z_i \) be three distinct vertices of \( G_i \). Let \( G \) be the graph obtained from the disjoint union of \( G_1, G_2, G_3 \) by adding the following edges: \( x_1x_2, x_2x_3, x_3x_1, y_1y_2, y_2y_3, y_3y_1 \). Assume \( G \) is 4-colorable. Show that \( G \) has a 4-coloring in which \( z_1, z_2, z_3 \) receive pairwise distinct colors.

2. Let \( G \) be a graph with a 4-flow. Show that there is a collection \( C \) of cycles in \( G \) such that each edge of \( G \) occurs in exactly two members of \( C \).

3. Prove that \( t_{r-1}(n) \leq \frac{1}{2}n^{2r-2}r^2 \), with equality iff \( r - 1 \) divides \( n \).

**NOTE:** There will be a 2 hour lecture on Friday, December 2. There will be no lectures on December 7 and 9.