HW 5 Chapter 5

8(b) $Tx = \begin{pmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix} \iff x_1 + x_2 + x_3 = 0$

$x = x_2 \begin{pmatrix} -1 \\ 1 \\ 0 \end{pmatrix} + x_3 \begin{pmatrix} -1 \\ 0 \\ 1 \end{pmatrix}$ $\operatorname{Ker} T = \operatorname{Span}\left\{ \begin{pmatrix} -1 \\ 1 \\ 0 \end{pmatrix}, \begin{pmatrix} -1 \\ 0 \\ 1 \end{pmatrix} \right\}$

For any $x \in \mathbb{R}^3$ $Tx = \begin{pmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} x_1 + x_2 + x_3 \\ x_1 + x_2 + x_3 \\ x_1 + x_2 + x_3 \end{pmatrix}$

$\operatorname{Range} T = \operatorname{span}\left\{ \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix} \right\}$

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10. Let λ be eigenvalue of A, let $\vec{v}_1$ $\vec{v}_3$ be linearly independent eigenvectors.

If $\exists \vec{v}_2 \neq 0$ $(A - \lambda I)\vec{v}_2 = \vec{v}_1$ $\exists \vec{v}_4 \neq 0$ $(A - \lambda I)\vec{v}_4 = \vec{v}_3$

Then $A \sim \begin{pmatrix} \lambda & 1 & 0 & 0 \\ 0 & \lambda & 0 & 0 \\ 0 & 0 & \lambda & 1 \\ 0 & 0 & 0 & \lambda \end{pmatrix}$

If $\exists \vec{v}_2 \neq 0$ $(A - \lambda I)\vec{v}_2 = \vec{v}_1$ $\exists \vec{v}_4 \neq 0$ $(A - \lambda I)\vec{v}_4 = \vec{v}_2$, then

$A \sim \begin{pmatrix} \lambda & 0 & 0 & 0 \\ 0 & \lambda & 1 & 0 \\ 0 & 0 & \lambda & 1 \\ 0 & 0 & 0 & \lambda \end{pmatrix}$

#13(b) $\mathcal{U}_2$ is open & dense

Open: for every $(x,y) \in \mathcal{U}_2$, $\exists$ a ball with a small radius r, s.t. $\left(|(x',y') - (x,y)| < r \Rightarrow (x',y') \in \mathcal{U}_2 \right)$ $B_r(x,y) \subset \mathcal{U}_2$

Dense: $\mathcal{U}_2$ intersects with every open set in $\mathbb{R}^2$

#15 (c) not generic