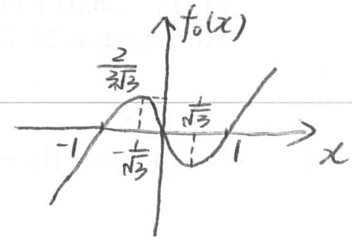
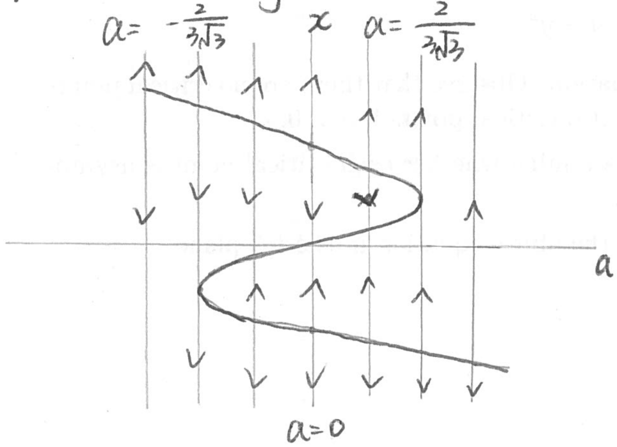
HW 1 Math 4541

Chapter 1 3(C), 4, 7, 14

3(C) Let $f_a(x) = x^3 - x + a$, graph of $f_0(x)$ is



Bifurcation diagram



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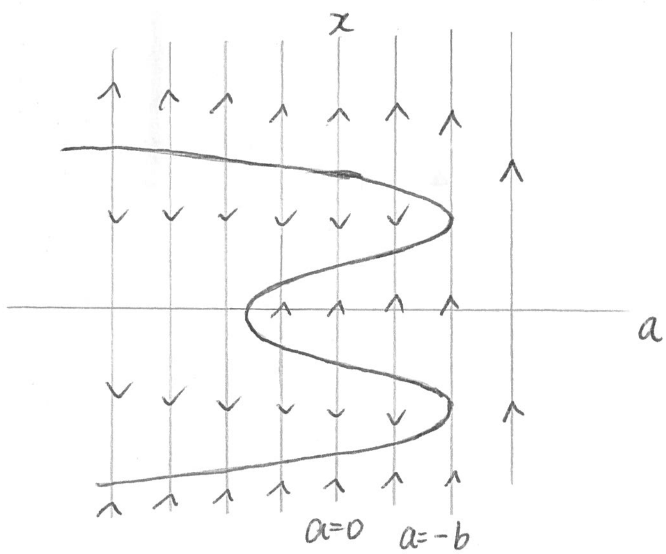
4. (a)

(b)



(c) Denote $f(0) = s$

When $a < -s$,

2 equilibrium pts

$a = -s$, saddle-node bifurcation

$-s < a < -b$ 4 equilibrium pts.

$a > -b$ no equilibrium pts

$a = -b$ two saddle-node bifurcation happen.

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7. $\begin{cases} x = 4 + c_1 e^{t} & t < 5 \\ x = 2 + c_2 e^{-t} & t \geq 5 \end{cases}$

(a) $x(0) = 4 \Rightarrow c_1 = 0 \Rightarrow x = 4$ for $t < 5$

$4 = x(5) = 2 + c_2 e^{-5} \Rightarrow c_2 = 2e^5$

$$x(t) = \begin{cases} 4, & t<5 \\ 2+2e^{5-t}, & t\geq 5 \end{cases}$$

$x(t)$ stays at 4 for $t<5$ then decreases and eventually approaches 2 for $t\to\infty$.

(b) $x(0)=3 \Rightarrow c_1=-1$ $\quad x(5)=4-e^5=2+c_2e^{-5}$

$\Rightarrow c_2 = 2e^5-e^{10}$

$$x(t) = \begin{cases} 4-e^t, & t<5 \\ 2+(2e^5-e^{10})e^{-t}, & t\geq 5 \end{cases}$$

$x(t)$ decreases for $t<5$, then increases and eventually tends to 2 for $t\to\infty$

(c) $t\to\infty$ all solutions tends to $x=2$ because 2 is a sink.

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14. $\frac{x'}{x}=p(t) \Rightarrow x(t)=x_0 e^{\int_0^t p(s)ds}$

where $x(0)=x_0$

$x(T+t)=x(t)$ for any x_0

$\Leftrightarrow$ $x_0 e^{\int_0^{T+t} p(s)ds} = x_0 e^{\int_0^t p(s)ds}$, $\forall x_0$

$\Leftrightarrow$ $e^{\int_0^{T+t} p(s)ds} = e^{\int_0^t p(s)ds}$

$\Leftrightarrow$ $\int_t^{T+t} p(s)ds = 0$

$\Leftrightarrow$ $\int_0^T p(t)dt=0$ due to periodicity

(Let $g(t)=\int_t^{T+t} p(s)ds$, $g'(t)=p(T+t)-p(t)=0$

$g(t)\equiv g(0)$)

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