Chap 2 #2(C) #6 #9 Chap 3 #1

#2(C) $\begin{vmatrix} 1-\lambda & 2 \\ 1 & -\lambda \end{vmatrix} = 0$ $\lambda_1 = 2$, $\lambda_2 = -1$

$v_1 = \begin{pmatrix} 2 \\ 1 \end{pmatrix}$, $v_2 = \begin{pmatrix} 1 \\ -1 \end{pmatrix}$

general solution $\vec{x}(t) = c_1 e^{2t} \begin{pmatrix} 2 \\ 1 \end{pmatrix} + c_2 e^{-t} \begin{pmatrix} 1 \\ -1 \end{pmatrix}$

#6 Characteristic equation $\lambda^2 + b\lambda + k = 0$

when $b^2 > 4k$ this system has distinct real eigenvalues

$\lambda_1 = \frac{-b+\sqrt{b^2-4k}}{2}$ $\lambda_2 = \frac{-b-\sqrt{b^2-4k}}{2}$

general solution $x(t) = c_1 e^{\frac{-b+\sqrt{b^2-4k}}{2}} + c_2 e^{\frac{-b-\sqrt{b^2-4k}}{2}}$

$\begin{cases} x(0) = 0 \\ x'(0) = 1 \end{cases} \Rightarrow c_1 = \frac{1}{\sqrt{b^2-4k}} = -c_2$

Solution satisfy initial condition is

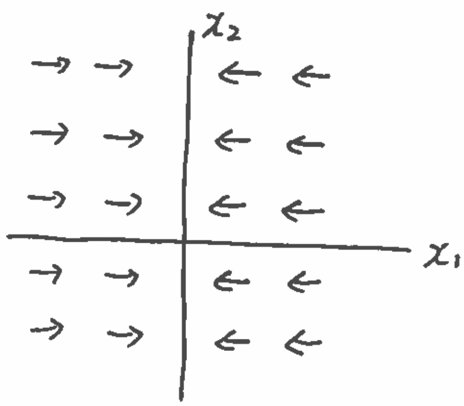
$x(t) = \frac{1}{\sqrt{b^2-4k}} e^{\frac{-b+\sqrt{b^2-4k}}{2}} - \frac{1}{\sqrt{b^2-4k}} e^{\frac{-b-\sqrt{b^2-4k}}{2}}$

Since $b \geq 0$ $k > 0$, then $\lambda_1 < 0$, $\lambda_2 < 0$ $x(t) \to 0$ as $t \to \infty$

#9 $x' = Ax$

$\begin{pmatrix} -e^{-t} \\ 0 \end{pmatrix} = A \begin{pmatrix} e^{-t} \\ \alpha \end{pmatrix} \Rightarrow A = \begin{pmatrix} -1 & 0 \\ 0 & 0 \end{pmatrix}$

general solution $\vec{x}(t) = c_1 e^{-t} \begin{pmatrix} 1 \\ 0 \end{pmatrix} + c_2 \begin{pmatrix} 0 \\ 1 \end{pmatrix}$



Chap3 #1

(a) $\lambda = \frac{1 \pm \sqrt{15}i}{2}$

(b) $\lambda = \frac{-1 \pm \sqrt{15}i}{2}$

(c) $\lambda = \frac{1 \pm \sqrt{15}i}{2}$

(d) $\lambda = \pm i$

(e) $\lambda = \pm i$

(f) $\lambda = \frac{-1 \pm \sqrt{15}i}{2}$

(a) ~ 3 (check direction field in a quadrant)

(b) ~ 5

(c) ~ 6

(d) ~ 1

(e) ~ 2

(f) ~ 4