

Chapter 4

#2 (d)
$$\begin{vmatrix} -\lambda & 0 & 2 \\ 0 & 2-\lambda & 0 \\ -2 & 0 & -\lambda \end{vmatrix} = 0 \Rightarrow (2-\lambda)(\lambda^2+4)=0$$

$$\Rightarrow \lambda_1=2, \lambda_2=2i, \lambda_3=-2i$$

$\lambda_1=2, \begin{pmatrix} -2 & 0 & 2 \\ 0 & 0 & 0 \\ -2 & 0 & -2 \end{pmatrix} \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix} \Rightarrow x=z=0 \Rightarrow \vec{v}_1 = \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix}$

$\lambda_2=2i, \begin{pmatrix} -2i & 0 & 2 \\ 0 & 2-2i & 0 \\ -2 & 0 & -2i \end{pmatrix} \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix} \Rightarrow \begin{matrix} -2ix+2z=0 \\ y=0 \end{matrix} \Rightarrow \vec{v}_2 = \begin{pmatrix} 1 \\ 0 \\ i \end{pmatrix}$

$\lambda_3=-2i, \vec{v}_3 = \begin{pmatrix} 1 \\ 0 \\ -i \end{pmatrix}$

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#6. When 2 has algebraic multiplicity 3

$$\begin{pmatrix} 2 & 0 & 0 & 0 & 0 \\ 0 & 2 & 0 & 0 & 0 \\ 0 & 0 & 2 & 0 & 0 \\ 0 & 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & -1 & 1 \end{pmatrix} \text{ or } \begin{pmatrix} 2 & 1 & 0 & 0 & 0 \\ 0 & 2 & 0 & 0 & 0 \\ 0 & 0 & 2 & 0 & 0 \\ 0 & 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & -1 & 1 \end{pmatrix} \text{ or } \begin{pmatrix} 2 & 1 & 0 & 0 & 0 \\ 0 & 2 & 1 & 0 & 0 \\ 0 & 0 & 2 & 0 & 0 \\ 0 & 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & -1 & 1 \end{pmatrix}$$

When 2 has algebraic multiplicity 1

$$\begin{pmatrix} 2 & 0 & 0 & 0 & 0 \\ 0 & 1 & 1 & 0 & 0 \\ 0 & -1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & -1 & 1 \end{pmatrix} \text{ or } \begin{pmatrix} 2 & 0 & 0 & 0 & 0 \\ 0 & 1 & 1 & 1 & 0 \\ 0 & -1 & 1 & 0 & 1 \\ 0 & 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & -1 & 1 \end{pmatrix}$$

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#7. L elementary matrix

L is identity matrix with i th, j th row interchanged

$\det L = -\det(I) = -1$

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