#1 (C)

HW6 Chapter 6

$$\begin{vmatrix} -\lambda & 1 & 0 \\ -1 & -\lambda & 0 \\ 1 & 1 & 1-\lambda \end{vmatrix} = 0 = -(\lambda^2+1)(\lambda-1)$$

$\Rightarrow \lambda_1 = 1 \quad \lambda_2 = i \,, \lambda_3 = -i$

For $\lambda_1 = 1$

$$\begin{pmatrix} -1 & 1 & 0 \\ -1 & -1 & 0 \\ 1 & 1 & 0 \end{pmatrix}\begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix} \Rightarrow \begin{matrix} x_1 = x_2 \\ x_1 + x_2 = 0 \end{matrix} \Rightarrow \vec{v}_1 = \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}$$

For $\lambda_2 = i$

$$\begin{pmatrix} -i & 1 & 0 \\ -1 & -i & 0 \\ 1 & 1 & 1-i \end{pmatrix}\begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix} \Rightarrow \begin{cases} x_2 = i x_1 \\ x_1 + x_2 + (1-i) x_3 = 0 \end{cases} \Rightarrow x_3 = -i x_1$$

$$\vec{v} = \begin{pmatrix} 1 \\ i \\ -i \end{pmatrix} = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} + i \begin{pmatrix} 0 \\ 1 \\ -1 \end{pmatrix}$$

$$e^{it}\vec{v} = (\cos t + i \sin t)\left[\begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} + i \begin{pmatrix} 0 \\ 1 \\ -1 \end{pmatrix}\right]$$

$$= \begin{pmatrix} \cos t \\ -\sin t \\ \sin t \end{pmatrix} + i \begin{pmatrix} \sin t \\ \cos t \\ -\cos t \end{pmatrix}$$

general sol'n $\quad \vec{x}(t) = c_1 e^t \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} + c_2 \begin{pmatrix} \cos t \\ -\sin t \\ \sin t \end{pmatrix} + c_3 \begin{pmatrix} \sin t \\ \cos t \\ -\cos t \end{pmatrix}$

#5.

(a) $\begin{vmatrix} -\lambda & 0 & a \\ 0 & b-\lambda & 0 \\ a & 0 & -\lambda \end{vmatrix} = 0 = (b-\lambda)(\lambda^2-a^2)$

$\Rightarrow \lambda_1 = b \quad \lambda_2 = a \quad \lambda_3 = -a$

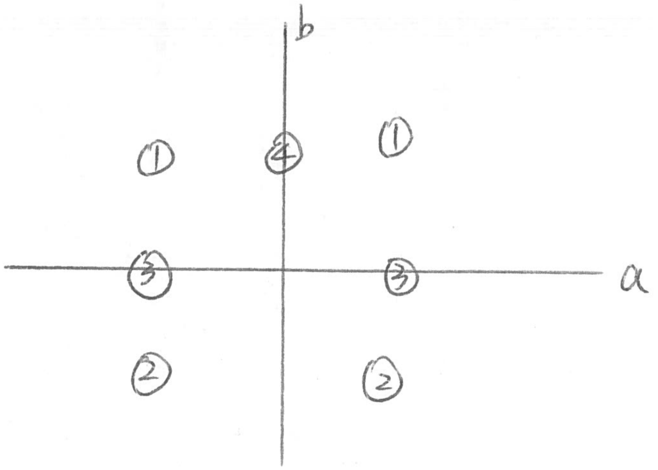
For $\lambda_1 = b \Rightarrow$ $\begin{pmatrix} -b & 0 & a \\ 0 & 0 & 0 \\ a & 0 & -b \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix} \Rightarrow$ $x_1 = x_3 = 0 \Rightarrow \vec{V}_1 = \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix}$

For $\lambda_2 = a \Rightarrow \begin{pmatrix} -a & 0 & a \\ 0 & b-a & 0 \\ a & 0 & -a \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix} \Rightarrow$ $x_2 = 0$, $-ax_1 + ax_3 = 0$ $\Rightarrow \vec{V}_2 = \begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix}$

For $\lambda_3 = -a$ $\begin{pmatrix} a & 0 & a \\ 0 & b+a & 0 \\ a & 0 & a \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix} \Rightarrow$ $x_2 = 0$, $+ax_1 + ax_3 = 0$ $\Rightarrow \vec{V}_3 = \begin{pmatrix} 1 \\ 0 \\ -1 \end{pmatrix}$

general sol'n $\vec{x}(t) = c_1 e^{bt} \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix} + c_2 e^{at} \begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix} + c_3 e^{-at} \begin{pmatrix} 1 \\ 0 \\ -1 \end{pmatrix}$

(b)



① $b>0 \quad a \neq 0$

2 unstable eigenspaces

1 stable eigenspace

② $b<0 \quad a \neq 0$

2 stable eigenspaces

1 unstable eigenspace

③ $b=0$ a line of equilibria

④ $a=0 \quad b>0$ plane of equilibria (other sol'ns go away from plane)

⑤ $a=0 \quad b<0$ 〃 (other sol'n tend to plane)

#11.

(a) $x_6 = 0$: $\Rightarrow$ $x_6 \equiv 0$

Along x_5 direction, sol'n dends to zero as $t \to +\infty$. Also, we have two harmonic oscillators in x_1, x_2/ x_3, x_4 space, which oscillate quasiperiodically. No periodic sol'n except for $\vec{x} = \vec{0}$

(b) $x_5 = 0$ $\Rightarrow$ $x_5 \equiv 0$

Along x_6 direction. sol'n tends to $+\infty / -\infty$ as $t \to +\infty$

The motion in x_1, x_2, x_3, x_4 space remain the same as (a).

(c) $x_3 = x_4 = x_5 = 0$. $\Rightarrow$ $x_3 \equiv x_4 \equiv x_5 \equiv 0$

Along x_6 direction, $t \to +\infty$ the sol'n goes far away from zero. In x_1, x_2 space, we have a harmonic oscillator with periodic motion.

(d) $x_3 = x_4 = x_5 = x_6 = 0 \Rightarrow$ $x_3 \equiv x_4 \equiv x_5 \equiv x_6 = 0$

In $x_1 x_2$ space, periodic motion as in (c)