

Let  $f \in C^0(\mathbb{R}^2, \mathbb{R})$  and consider

$$x' = f(t, x), \quad x(0) = x_0.$$

Suppose there exist 2 solution  $x_1(t)$  and  $x_2(t)$  on  $t \in [0, 1]$  and  $x_2(1) > x_1(1)$ . Show that for any  $\tilde{x} \in (x_1(1), x_2(1))$ , there exists a solution  $x(t)$  to the above initial value problem such that  $x(1) = \tilde{x}$ .