• **1.** Let X be a complete metric space and $f : X \to X$. Suppose $\lambda \in (0,1)$, $n \in \mathbb{N}$, and f satisfy

$$d\big(f^{(n)}(x), f^{(n)}(y)\big) \leq \lambda d(x,y), \quad \forall x, y \in X.$$

Are there any fixed points of f? If so, unique?

• **2.** Let X and Y be Banach spaces and $T : X \to Y$ an isomorphism (linear, continuous, with continuous linear inverse $T^{-1} : Y \to X$). $g : X \to Y$ and $g(0) = 0$.

(1) Suppose g is continuous and $\exists C > 0$ such that $|g(x)| \leq C$, $\forall x \in X$. Does there exists $C_0 > 0$ such that $f(x) = Tx + g(x)$ is invertible whenever $C < C_0$? If so, is the inverse always continuous? Prove your conclusion or give a counterexample.

(2) Suppose g is Lipschitz, i.e. $\exists$ Lipschitz constant $L > 0$ such that

$$|g(x) - g(y)| \leq L|x - y|, \quad \forall x, y \in X.$$

Does there exists $L_0 > 0$ such that $f(x) = Tx + g(x)$ is invertible whenever $L < L_0$? If so, is the inverse always Lipschitz (with some Lipschitz constant)? Prove your conclusion or give a counterexample.