P72. 5(a) HW1

Let $A = \begin{pmatrix} -1 & 1 \\ 0 & 2 \end{pmatrix}$ $B = \begin{pmatrix} 1 & 0 \\ 1 & -2 \end{pmatrix}$

If $P = \begin{pmatrix} 1 & 1 \\ 0 & 3 \end{pmatrix}$ $P^{-1} = \begin{pmatrix} 1 & -\frac{1}{3} \\ 0 & \frac{1}{3} \end{pmatrix}$, then

$$P^{-1}AP = \begin{pmatrix} -1 & 0 \\ 0 & 2 \end{pmatrix}$$

If $Q = \begin{pmatrix} 0 & 3 \\ 1 & 1 \end{pmatrix}$ $Q^{-1} = \begin{pmatrix} -\frac{1}{3} & 1 \\ \frac{1}{3} & 0 \end{pmatrix}$ then

$$Q^{-1}BQ = \begin{pmatrix} -2 & 0 \\ 0 & 1 \end{pmatrix}$$

Define $h_1(x) = \begin{cases} x^2, & x \geq 0. \\ -x^2, & x < 0. \end{cases}$

$h_2(\cancel{x}\ y) = \begin{cases} \sqrt{y}, & y \geq 0, \\ -\sqrt{|y|}, & y < 0. \end{cases}$

$h(x,y) = (h_1(x), h_2(y))$

Then $\phi_B(t, Q \circ h \circ P^{-1}(x_0)) = Q \circ h \circ P^{-1}(\phi_A(t, x_0))$

$H = Q \circ h \circ P^{-1}$ is our conjugacy. #

P136 #6

(a) eigenvalues $\lambda_1 = b$, $\lambda_2 = a+bi$, $\lambda_3 = a-bi$

eigenvectors $\vec{v}_1 = \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix}$ $\vec{v}_2 = \begin{pmatrix} 1 \\ 0 \\ i \end{pmatrix}$

general sol'n

$$\vec{X}(t) = c_1 e^{bt} \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix} + c_2 e^{at} \begin{pmatrix} \cos bt \\ 0 \\ -\sin bt \end{pmatrix} + c_3 e^{at} \begin{pmatrix} \sin bt \\ 0 \\ \cos bt \end{pmatrix}$$

(b) Let $X = \begin{pmatrix} x \\ y \\ z \end{pmatrix}$

$b<0$, $a<0$ (0,0,0) is asym. stable
stable in y direction, spiral sink in xz-plane

$a=0$ (0,0,0) is stable not asym. stable
a collection of periodic orbits in xz-plane

$a>0$ (0,0,0) is unstable
spiral source in xz-plane

$b>0$, $a<0$, $a=0$, $a>0$: similar behavior in xz-plane as above
(0,0,0) is unstable in all these cases

$b=0$, $a>0$ (0,0,0) is unstable
$a<0$ (0,0,0) is stable but not asym. stable

In above two cases, static in y direction, no spirals in xz-plane

$b=0$ $a=0$ every point in $\mathbb{R}^3$ is equilibrium point

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P138 #15

The equations are equivalent to

$$\begin{pmatrix} x_1'(t) \\ y_1'(t) \\ x_2'(t) \\ y_2'(t) \\ x_3'(t) \\ y_3'(t) \end{pmatrix} = \begin{pmatrix} \begin{matrix} 0 & 1 \\ -1 & 0 \end{matrix} & 0 & 0 \\ 0 & \begin{matrix} 0 & \sqrt{2} \\ -\sqrt{2} & 0 \end{matrix} & 0 \\ 0 & 0 & \begin{matrix} 0 & \omega \\ -\omega & 0 \end{matrix} \end{pmatrix} \begin{pmatrix} x_1(t) \\ y_1(t) \\ x_2(t) \\ y_2(t) \\ x_3(t) \\ y_3(t) \end{pmatrix}$$

If $\frac{w}{\sqrt{2}}$ is rational. we have periodic motions in x_2 and x_3, quasiperiodic everywhere else.

If $\frac{w}{\sqrt{2}}$ is irrational, quasiperiodic motions everywhere

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