HW3 Chapter 9

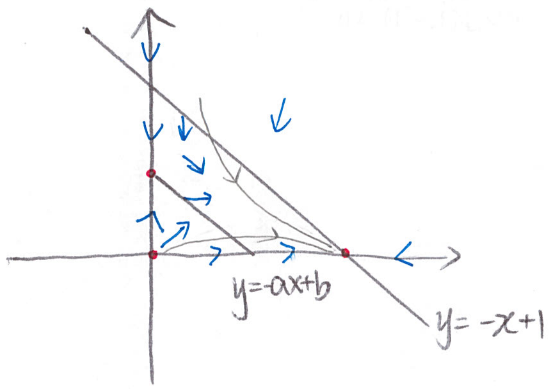
#3 $x'=0 \Rightarrow x=0$ or $y=-x+1$

$y'=0 \Rightarrow y=0$ or $y=-ax+b$

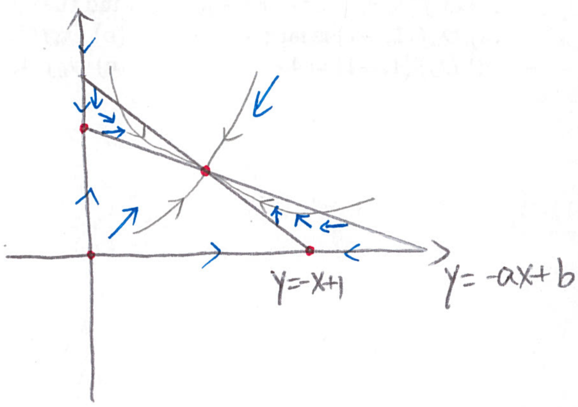
We'll always have $(0,0)$ $(0,b)$ $(1,0)$ as our equilibr

If $y=-x+1$ intersect with $y=-ax+b$ inside first quadrant, we'll have one more equilibrium.

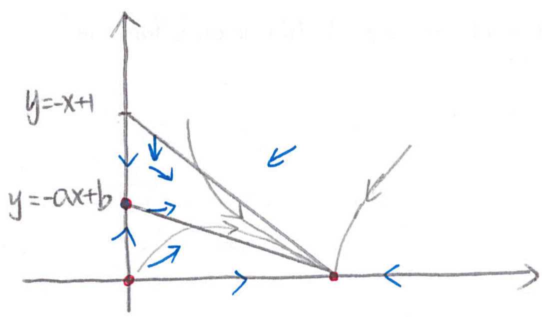
I: $b<1$, $\frac{b}{a}<1$



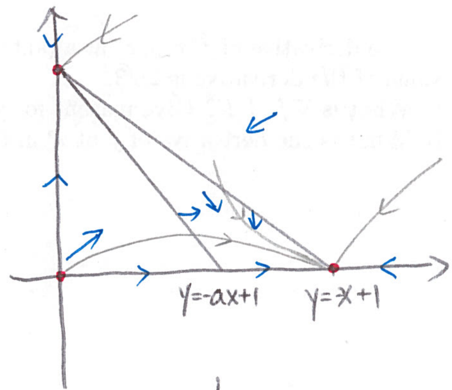
II $b<1$ $\frac{b}{a}>1$



III $b<1$ $b=a$

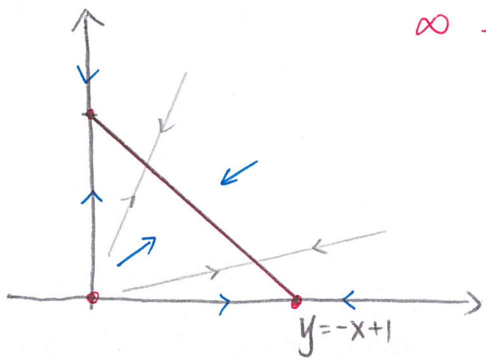


IV $b=1$ $\frac{b}{a}<1$

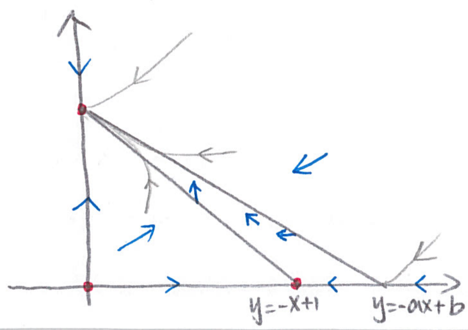


V $a=b=1$

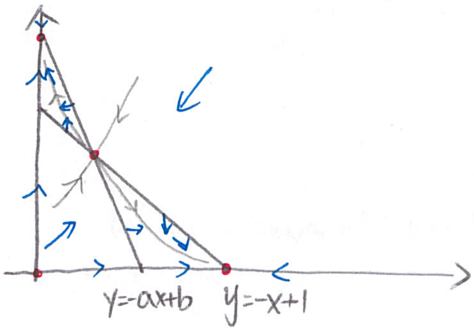
∞ equilibria



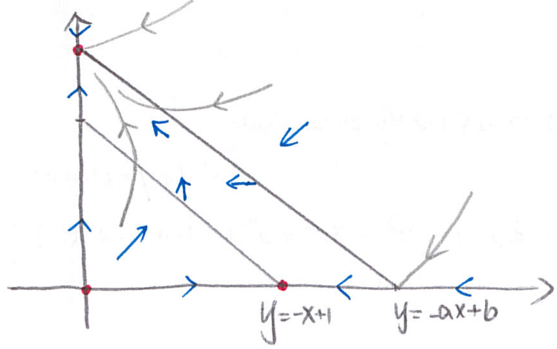
VI $b=1$ $\frac{b}{a}>1$



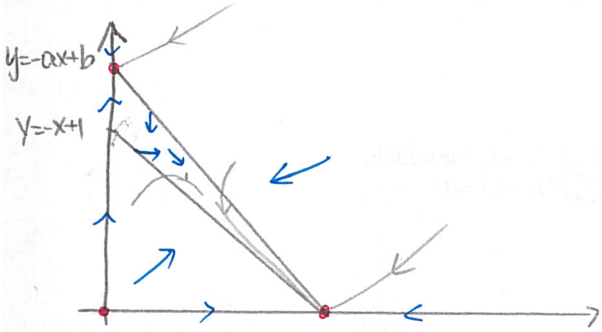
VII $b>1$ $\frac{b}{a}<1$



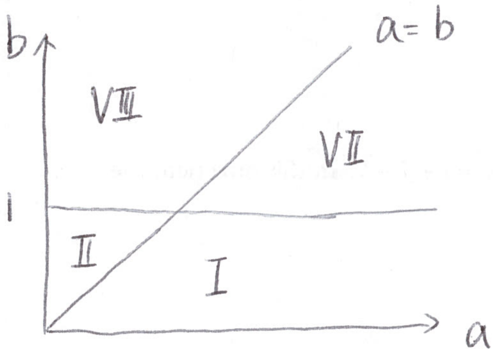
VIII $b>1$ $\frac{b}{a}>1$



IX $b>1$ $a=b$



Bifurcation occurs when $b=1$, $a=b$.



#6. Let $L(x,y) = x^2+y^2$, the $L(0,0) = 0$

$$\dot{L} = 2xx' + 2yy' = 2x(-2x-y^2) + 2y(-y-x^2)$$

$$= -2\left(2x^2 + xy^2 + y^2 + yx^2\right)$$

$$= -2\left[(2+y)x^2 + (x+1)y^2\right]$$

$\dot{L} < 0$ if $x > -1$ and $y > -2$.

If we consider polar coordinates $x = r\cos\theta$, $y = r\sin\theta$

$$\dot{L} = -2\left(2r^2\cos^2\theta + r^2\sin^2\theta + r^3\cos\theta\sin^2\theta + r^3\cos^2\theta\sin\theta\right)$$

$$= -2r^2\left(1+\cos^2\theta + r\cos\theta\sin^2\theta + r\cos^2\theta\sin\theta\right)$$

$$= -2r^3\left(1+\cos^2\theta\right)\left(\frac{1}{r} + \frac{\cos\theta\sin\theta(\cos\theta+\sin\theta)}{1+\cos^2\theta}\right)$$

since

$$\frac{\cos\theta\sin\theta(\cos\theta+\sin\theta)}{1+\cos^2\theta} > -0.495$$

If $r < \frac{1}{0.495} \approx 2.02$

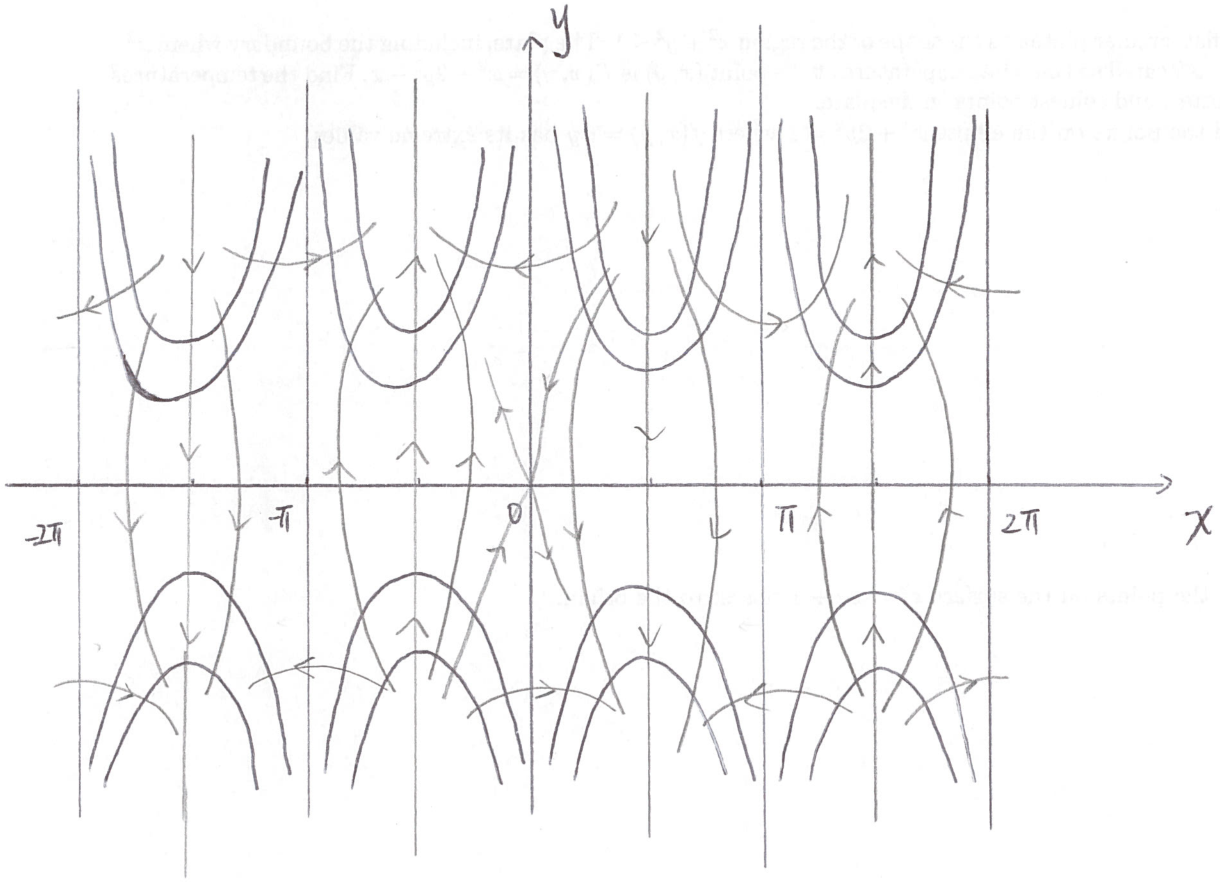
Then $\dot{L} < 0$. Let $\delta = 2.02$, $B(0,\delta)$ is contained in the basin of attraction. #

#7 (C)

$$\begin{cases} x' = -y\cos x \\ y' = -\sin x \end{cases}$$

$V(x,y) = y\sin x = C$

$y\sin x = 0 \Rightarrow y = 0$ or $x = k\pi, \ k \in \mathbb{Z}$



$-\sin x = 0 \Rightarrow x = k\pi$ equilibria $(k\pi, 0)$

$y\cos x = 0 \Rightarrow y = 0$

Linearized equation

$$x' = \begin{pmatrix} 0 & -\cos(k\pi) \\ -\cos(k\pi) & 0 \end{pmatrix} \begin{pmatrix} x - k\pi \\ y \end{pmatrix}$$

$\cos k\pi = \pm 1$ $\quad \det \begin{pmatrix} 0 & -\cos(k\pi) \\ -\cos(k\pi) & 0 \end{pmatrix} = -\cos^2(k\pi) = -1 < 0$

saddle type equilibria.

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