1.6 Show that the mixed partial derivatives of the function

\[ f(x, y) = \begin{cases} 
\frac{x^3 y - x y^3}{x^2 + y^2} & \text{if } (x, y) \neq (0, 0) \\
0 & \text{if } (x, y) = (0, 0)
\end{cases} \]

are not equal. Explain.

For \((x, y) \neq (0, 0)\):

\[ f_x(x, y) = \frac{(3x^2 y - x y^3) (x^2 + y^2) - (2x)(x^3 y - x y^3)}{(x^2 + y^2)^2} = \frac{x^4 y + 4x^2 y^3 - y^5}{(x^2 + y^2)^2} \]

\[ f_y(x, y) = \frac{(x^3 - 3xy^2) (x^2 + y^2) - (2y)(x^3 y - x y^3)}{(x^2 + y^2)^2} = \frac{x^5 - 4x^3 y^3 - xy^4}{(x^2 + y^2)^2} \]

\[ f_x(0, y) = -y \quad \Rightarrow \quad f_{yx}(0, y) = -1. \quad \text{If } x \neq 0, \quad f_{xy}(x, 0) = 1 \]

\[ f_y(x, 0) = x \quad \Rightarrow \quad f_{yx}(0, 0) = 1. \quad \text{If } y \neq 0, \quad f_{xy}(0, y) = -1 \]

\text{Explanation: } \text{On } \mathbb{R}^2, \text{it says } f_{xy} = f_{yx} \text{ if they are continuous.}
\text{Here they are not, so } f_{xy} \text{ and } f_{yx} \text{ don't have to be equal.}

2.4. In \(\mathbb{R}^2\), a level surface of a function \(F\) of two variables is a curve (level curve of \(F\)) and a tangent plane is a line. Find the equation of the line tangent to the curve

\[ x^4 + x^3 y^3 + y^4 = 21 \]

at the point \((1, 2)\).

Let \(F(x, y) = x^4 + x^3 y^3 + y^4\)

\[ \frac{\partial F}{\partial x} = 4x^3 + 2x^2 y^3 \quad \frac{\partial F}{\partial y} = 2x^3 y + 4y^3 \]

\[ \nabla F(x, y) = \langle 4x^3 + 2x^2 y^3, 2x^3 y + 4y^3 \rangle \]

\[ \nabla F(1, 2) = \langle 12, 26 \rangle \]

So the tangent line is \(12(x - 1) + 26(y - 2) = 0 \quad \Rightarrow \quad y = -\frac{1}{3} x + \frac{2}{3} \)
3.5. Find a parametric representation for the space curve given by
\[ x^2 + y^2 = 1, \quad x + y + z = 0. \]

\[ \cos^2 \theta + \sin^2 \theta = 1 \quad \Rightarrow \quad x = \cos \theta, \quad y = \sin \theta \]

\[ z = -\cos \theta - \sin \theta \quad \theta \in [0, 2\pi]. \]