HW #2.

Sec. 1.4: 4.4 (b) Solve \( t \frac{dx}{dt} = \frac{x}{x^2 + 1} \)

Solu: \( \int \frac{x^2 + 1}{x} \, dx = \int t \, dt \Rightarrow \log|x| + C = \frac{1}{2} x^2 + \log|t| \)

\( \Rightarrow \log|x| = \log|t| + C \) \( \Rightarrow x e^{\frac{1}{2}x^2} = C t \)

Implicit form of solu.

Sec. 2.1: 1.7 C: \( x = x(t), \ y = y(t), \ z = z(t) \), is an integral curve of \( \vec{X} \) and \( u(x, y, z) \).

Proof: \( \frac{d}{dt} u(x(t), y(t), z(t)) = \nabla u \cdot (\dot{x(t)}, \dot{y(t)}, \dot{z(t)}) \)

\( \Rightarrow u(t) = 0 \) since \( u \) is a first integral.

\( \Rightarrow \) \( U \) is a constant along \( C \).

C lies on a single level surface of \( U \).

Sec. 2.2: 2.8 (a) Find general solu \( \frac{dx}{dy - z} = \frac{dt}{2 - x} = \frac{dz}{x - y} \).

Solu: let \( V = x + y + z \). \( \Rightarrow \)

\( dv = dx + dy + dz = (y - z) \, dt + (2 - x) \, dt + (x - y) \, dt \)

\( \Rightarrow u_1(x, y, z) = x + y + z \) is a first integral.

so a const along any integral curves \( z = u_1 - x \).

\( \frac{dx}{y - z} = \frac{dy}{2 - x} \Rightarrow \frac{dx}{x + 2y - u_1} = \frac{dy}{u_1 - 2x - y} \)

\( f(u_1, 2x + y) \, dx + (x + 2y - u_1) \, dy = 0 \Rightarrow d(x^2 + y^2 + \frac{b^2}{4} - (x + \alpha)(x + y + z)) = 0 \)

\( U_2(x, y, z) = x^2 + y^2 + \frac{b^2}{4} - (x + \alpha)(x + y + z) \)

\(-x^2 - y^2 - z^2 \). is also a first integral.
Sec. 2.4. 4.16. \( \vec{U} = (x, -y, 0) \) find integral surface \( C : x = t, \quad y = t, \quad z = t^2, \quad t > 0 \) containing \( C \).

**Solution:** Solve for \((x(t, s), y(t, s), z(t, s))\)

\[
\begin{align*}
x_s &= x & \text{with } x(t, 0) &= t \\
y_s &= -t & \text{with } y(t, 0) &= t \\
z_s &= 0 & \text{with } z(t, 0) = t^2
\end{align*}
\]

\( \Rightarrow \) \( x(t, s) = t e^s \), \( y(t, s) = t e^s \), \( z(t, s) = t^2 \).

Let \( S \) be the surface with the above parametric eqn. \( S \) works.

**RK (a) the above:** \( S : xy - z = 0 \)

(b) Another method:

Step 1: Find (work!) first integrals:

\( U_1(x, y, z) = x \frac{d}{dt} \), \( U_2(x, y, z) = z \).

Step 2: \( U_1(t) = U_1(x(t, t^2), y(t, t^2), z(t, t^2)) = t^2 \).

\( U_2(t) = U_2(\ldots) = t^2 \).

\( \Rightarrow t = 2U_1 \) and \( U_2 - U_1 = 0 \) \( \Rightarrow \)

\( U(x, y, z) = U_1(x, y, z) - U_2(x, y, z) = xy - z \) is a first integral and \( S : U = xy - z = 0 \) works.