HW # 1

0.2 #19

$$t^2 \frac{d^2u}{dt^2} + t\frac{du}{dt} - u = 1. \qquad u_1(t) = t \qquad u_2 = 1/t$$

$$\Leftrightarrow \frac{d^2u}{dt^2} + \frac{1}{t}\frac{du}{dt} - \frac{1}{t^2}u = \frac{1}{t^2}$$

Wronskian: $W = \begin{vmatrix} t & \frac{1}{t} \\ 1 & -\frac{1}{t^2} \end{vmatrix} = -\frac{2}{t}$

$$\Rightarrow v_1(t) = -\int \frac{u_2(t) f(t)}{W(t)} dt = -\int \frac{\frac{1}{t}\cdot\frac{1}{t^2}}{-\frac{2}{t}} dt = -\frac{1}{2}\frac{1}{t}$$

$$v_2(t) = \int \frac{u_1(t) f(t)}{W(t)} dt = \int \frac{t\cdot\frac{1}{t^2}}{-\frac{2}{t}} dt = -\frac{1}{2}t$$

$$\Rightarrow u_p(t) = v_1(t)\, u_1(t) + v_2(t)\, u_2(t) = -1$$

0.3. 6.

$$\frac{d^2u}{dx^2} - \frac{T}{EI}u = -\frac{\omega}{EI}\frac{Lx - x^2}{2} \qquad 0 < x < L$$

$$u(0) = 0 \quad , \quad u(L) = 0$$

General solution for the homogeneous equation is:

$$u_h(x) = C_1\cosh(\lambda x) + C_2\sinh(\lambda x)$$

where $\lambda = \sqrt{\frac{T}{EI}}$

Assume particular solution is in the form of

$$U_p(x) = ax^2 + bx + c$$

Plug $U_p(x)$ back into equation and compare coefficients of x^2, x and 1:

$$a = -\frac{w}{2T}.$$

$$b = \frac{wL}{2T}$$

$$c = -\frac{wEI}{T^2}$$

So: $U_p(x) = -\frac{w}{2T}x^2 + \frac{wL}{2T}x - \frac{wEI}{T^2}$

And general solution is:

$$U(x) = U_h(x) + U_p(x). \quad \cdots \quad (*)$$

Use boundary condition:

$$U(0) = 0 \Rightarrow C_1 = \frac{wEI}{T^2}$$

$$U(L) = 0 \Rightarrow$$

$$\frac{wEI}{T^2}\cosh(\lambda L) + C_2\sinh(\lambda L) - \frac{wEI}{T^2} = 0$$

$$\Rightarrow C_2 = \frac{wEI}{T^2}\,\frac{1-\cos(\lambda L)}{\sinh(\lambda L)}$$

Plug back to $(*)$ and get solution

0.4.4

$$\frac{\lambda}{3}\frac{d}{d\rho}\left(\rho^2\frac{du}{d\rho}\right) + (k-1)A\rho^2 u = 0$$

$$\Leftrightarrow \quad \frac{d}{d\rho}\left(\rho^2\frac{du}{d\rho}\right) + \frac{3(k-1)A}{\lambda}\rho^2 u = 0$$

let $\mu^2 = \dfrac{3(k-1)A}{\lambda}$. then the equation is

$$\frac{d}{d\rho}\left(\rho^2\frac{du}{d\rho}\right) + \mu^2\rho^2 u = 0 \quad \cdots \quad (*)$$

let $u = \dfrac{v}{\rho}$.

$$\Rightarrow \quad \frac{du}{d\rho} = \frac{\rho v' - v}{\rho^2} \quad \Rightarrow \quad \rho^2\frac{du}{d\rho} = \rho v' - v$$

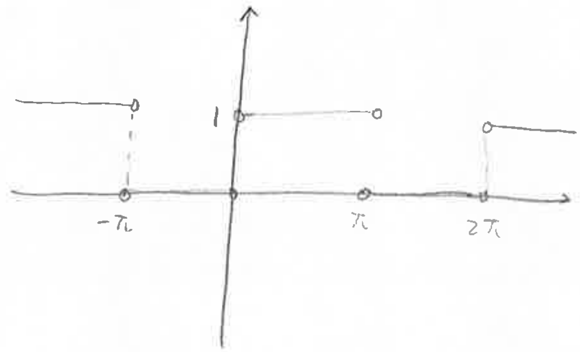
$$\Rightarrow \quad \frac{d}{d\rho}\left(\rho^2\frac{du}{d\rho}\right) = v' + \rho v'' - v' = \rho v''$$

$$\Rightarrow \quad (*) \Leftrightarrow \rho v'' + \mu^2\rho v = 0 \quad \Leftrightarrow \quad v'' + \mu^2 v = 0$$

1.1.(1c).

$$f(x) = \begin{cases} 0 & -\pi < x < 0 \\ 1, & 0 < x < \pi \end{cases} \qquad 2\pi\text{- periodic.}$$

Then:



$$a_0 = \frac{1}{2\pi}\int_{-\pi}^{\pi} f(x)\,dx$$

$$= \frac{1}{2\pi}\cdot\pi = \frac{1}{2}$$

$$a_n = \frac{1}{\pi}\int_{-\pi}^{\pi} f(x)\cos(mx)\,dx$$

$$= \frac{1}{\pi}\int_{0}^{\pi}\cos(mx)\,dx = 0 \qquad \text{for } n \neq 0.$$

$$b_n = \frac{1}{\pi}\int_{-\pi}^{\pi} f(x)\sin(nx)\,dx$$

$$= \frac{1}{\pi}\int_{0}^{\pi}\sin(nx)\,dx = \frac{1}{n\pi}(1-\cos n\pi) = \begin{cases} 0. & \text{for } n \text{ even} \\ \frac{2}{\pi}\cdot\frac{1}{n} & \text{for } n \text{ odd} \end{cases}$$

$\Rightarrow$ Fourier series:

$$\frac{1}{2} + \frac{2}{\pi}\left(\sin x + \frac{1}{3}\sin 3x + \frac{1}{5}\sin 5x + \dots\right)$$