Sec 0.2, 19

$$t^2\frac{d^2u}{dt^2} + t\frac{du}{dt} - u = 1 \qquad u_1(t) = t, \quad u_2(t) = \frac{1}{t}$$

$$\frac{d^2u}{dt^2} + \frac{1}{t}\frac{du}{dt} + \left(\frac{-1}{t^2}\right)u = \frac{1}{t^2} \rightarrow \text{standard form}$$

corrs. homogeneous equation is

$$\frac{d^2u}{dt^2} + \frac{1}{t}\cdot\frac{du}{dt} + \left(\frac{-1}{t^2}\right)u = 0$$

assume the sol^n is of the form:

$$u_p(t) = v_1 u_1(t) + v_2 u_2(t)$$

$$u_p(t) \quad v_1 t + \frac{v_2}{t}$$

The pairs of simultaneous equations are:-

$$v_1' u_1 + v_2' u_2 = 0$$

$$v_1' u_1' + v_2' u_2' = f(t)$$

Hence,

$$v_1'\cdot t + v_2'\cdot\frac{1}{t} = 0 \quad — (1)$$

$$v_1' 1 + v_2'\left(-\frac{1}{t^2}\right) = \frac{0}{t^2} \quad — (2).$$

$$v_1' t + \frac{v_2'}{t} = 0 \quad — (1) \qquad v_1' - \frac{v_2'}{t^2} = \frac{0}{t^2} \quad — (2)$$

$$v_1' = -\frac{v_2'}{t^2}$$, substituting this in (2)

$$\cancel{\frac{-2v_2'}{t^2}} = 0 \qquad \cancel{v_2'} = 0$$

v_1

$$-\frac{v_2'}{t^2} - \frac{v_2'}{t^2} = \frac{1}{t^2}$$

$$v_2' = -\frac{1}{2}$$

$$v_2(t) = -\frac{1}{2}\int dt$$

$$v_2(t) = -\frac{1}{2}\cdot t$$

$$v_1' = -\frac{v_2'}{t^2} = \frac{1}{2t^2}$$

$$v_1(t) = \frac{1}{2}\int \frac{1}{t^2}\,dt = \frac{-1}{2t}$$

$$u_p = v_1(t)\cdot u_1(t) + v_2(t)\cdot u_2(t)$$

$$= \frac{-1}{2t}\cdot t + \left(-\frac{1}{2}\cdot t\right)\cdot\frac{1}{t} = -1$$

$$\therefore u_p = -1$$

0.3 6. $$\frac{d^2u}{dx^2} - \frac{T}{EI}u = \frac{-w}{EI}\,\frac{Lx - x^2}{2} \qquad 0 < x < l \quad \text{— (1)}$$

$$u(0) = 0,\ u(L) = 0$$

$$\frac{d^2u}{dx^2} - \frac{T}{EI}u = 0 \Rightarrow u(x) = C_1 e^{\sqrt{\frac{T}{EI}}x} + C_2 e^{-\sqrt{\frac{T}{EI}}x} \quad \text{— (2)}$$

Let particular sol^n be

$$u(x) = Ax + Bx^2 + D \quad \text{— (3)}$$

plug in original eq^n. to get

$$A = \frac{wL}{2T}, \quad B = \frac{-w}{2T}, \quad D = -\frac{wEI}{T^2}$$

plug A, B, D in (3) and apply boundary values to get:

$$C_1 + C_2 = \frac{wEI}{T^2} \quad \text{— (a)}$$

$$C_1 e^{\sqrt{\frac{T}{EI}}\cdot L} + C_2 e^{\sqrt{\frac{T}{EI}}\cdot L} + \frac{wL^2}{2T} - \frac{wL^3}{2T} - \frac{wEI}{T^2} = 0 \quad \text{— (b)}$$

find C_1, C_2 by solving (a) and (b)

and plug the values in (3)

$$u(x) = C_1 e^{\sqrt{\frac{T}{EI}}x} + C_2 e^{\sqrt{\frac{T}{EI}}x} + \frac{wL}{2T}x - \frac{w}{2T}x^2 - \frac{wEI}{T^2} \quad \text{— (3)}$$