0.3.3b:-

The general solution is given by:

$$u(x) = c_1 \cos(\lambda x) + c_2 \sin(\lambda x)$$

$$\frac{du}{dx} = -c_1 \lambda \sin(\lambda x) + c_2 \lambda \cos(\lambda x)$$

Applying BV condition,

$$\frac{du}{dx}(0) = c_2 \lambda = 0$$

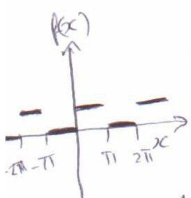
$$u(a) = c_1 \cos(\lambda a) + c_2 \sin(\lambda a) = 0$$

$$\Rightarrow \cos(\lambda a) = 0$$

$$\Rightarrow \lambda = \pm \frac{\pi}{2a}, \pm \frac{3\pi}{2a}, \pm \frac{5\pi}{2a} \dots$$

1.1.1c:-

$$a_0 = \frac{1}{2\pi} \int_{-\pi}^{\pi} f(x)\,dx = \frac{1}{2\pi} \int_0^{\pi} dx = \frac{1}{2\pi} \pi = \frac{1}{2}$$



$$a_n = \frac{1}{\pi} \int_{-\pi}^{\pi} f(x) \cos(nx)\,dx = \frac{1}{\pi} \int_0^{\pi} \cos(nx)\,dx = \frac{1}{\pi} \left[\frac{\sin(nx)}{n} \right]_0^{\pi} = 0$$

$$b_n = \frac{1}{\pi} \int_{-\pi}^{\pi} f(x) \sin(nx)\,dx = \frac{1}{\pi} \int_0^{\pi} \sin(nx)\,dx = \frac{1}{n\pi} \left(1 - (-1)^n\right)$$

$$f(x) = a_0 + \sum_{n=1}^{\infty} \left(b_n \sin(nx)\right) = \frac{1}{2} + \frac{1}{\pi} \sum_{n=1}^{\infty} \frac{1-(-1)^n}{n} \sin(nx)$$

1.2.3 :-

$$a_0 = \frac{1}{2a}\int_{-a}^{a} f(x)\,dx = \frac{1}{2a}\int_{0}^{2a} f(x)\,dx$$

$$a_n = \frac{1}{a}\int_{-a}^{a} f(x)\cos\left(\frac{n\pi x}{a}\right)dx = \frac{1}{a}\int_{0}^{2a} f(x)\cos\left(\frac{n\pi x}{a}\right)dx$$

$$b_n = \frac{1}{a}\int_{-a}^{a} f(x)\sin\left(\frac{n\pi x}{a}\right)dx = \frac{1}{a}\int_{0}^{2a} f(x)\sin\left(\frac{n\pi x}{a}\right)dx$$

1.2.11c :-

Odd Extension:-

$$b_n = \frac{2}{a}\int_{0}^{a} f(x)\sin\left(\frac{n\pi x}{a}\right)dx = 2\int_{0}^{1}\sin(x)\sin(n\pi x)\,dx$$

$$= 2\left(\left(\frac{\sin(1-n\pi)}{2(1-n\pi)}x\right)_0^1 - \left(\frac{\sin(1+n\pi)}{2(1+n\pi)}x\right)_0^1\right)$$

$$= \frac{\sin(n\pi-1)}{\cancel{2}(n\pi-1)} - \frac{\sin(n\pi+1)}{(n\pi+1)}$$

$$f(x) \sim \sum_{n=1}^{\infty}\left(\frac{\sin(n\pi-1)}{(n\pi-1)} - \frac{\sin(n\pi+1)}{(n\pi+1)}\right)\sin(n\pi x)$$

Even Extension :-

$$a_0 = \frac{1}{a}\int_0^a \sin(x)\,dx = -1\left(\cos(x)\right)_0^1 = 1 - \cos(1)$$

$$a_n = \frac{2}{a}\int_0^a \sin(x)\cos\left(\frac{n\pi x}{a}\right)dx = 2\int_0^1 \sin(x)\cos(n\pi x)\,dx$$

$$= \frac{\cos(1-n\pi)-1}{(1-n\pi)} - \frac{\cos(1+n\pi)-1}{(1+n\pi)}$$

$$f(x) \sim 1-\cos(1) + \sum_{n=1}^{\infty}\left[\frac{\cos(1-n\pi)-1}{(1-n\pi)} - \frac{\cos(1+n\pi)-1}{(1+n\pi)}\right]\cos(n\pi x)$$

1.3.7 :-

$$C = \frac{1}{4}$$

$$A = \frac{-\pi^2}{12}$$

$$\Rightarrow f(x) = \frac{-\pi^2}{12} + \frac{1}{4}x$$

1.4.2 :-

At $x=0$, $F(x)$ converges to 1.

At $x = \pm\pi$, $f(x)$ converges to 0.

$F(x)$ has uniform converges because of removable discontinuity at $x=0$.