

SOLUTIONS - ASSIGNMENT 3

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$$a) |x| = \frac{1}{2} - \frac{4}{\pi^2} \sum_{k=0}^{\infty} \frac{1}{(2k+1)^2} \cos((2k+1)\pi x), -1 < x < 1$$

$$\left[\frac{1}{2} - |x| \right] \frac{\pi^2}{4} = \sum_{k=0}^{\infty} \frac{1}{(2k+1)^2} \cos((2k+1)\pi x)$$

$$\frac{\pi^2}{8} = 1 + \frac{1}{9} + \frac{1}{25} + \dots$$

$$b) \frac{4}{\pi} \sum_{k=0}^{\infty} \frac{1}{(2k+1)} \sin((2k+1)\pi x) = \begin{cases} -1 & -1 < x < 0 \\ 1 & 0 < x < 1 \end{cases}$$

$$\sum_{k=0}^{\infty} \frac{1}{(2k+1)} \sin((2k+1)\pi x) = \begin{cases} -\pi/4 & -1 < x < 0 \\ \pi/4 & 0 < x < 1 \end{cases}$$

$$\frac{\pi}{4} = 1 - \frac{1}{3} + \frac{1}{5} - \frac{1}{7} + \dots$$

$$c) |\sin x| = \frac{2}{\pi} - \frac{4}{\pi} \sum_{n=2}^{\infty} \frac{1}{4n^2-1} \cos(2nx)$$

$$\left[\frac{1}{2} - |\sin(x)| \right] \frac{\pi}{4} = \sum_{n=2}^{\infty} \frac{1}{4n^2-1} \cos(2nx)$$

$$\frac{1}{2} = \frac{1}{3} + \frac{1}{15} + \frac{1}{35} + \dots$$