   2. Show your work and explain your answers and reasoning.
   3. Calculators may be used, but are by no means necessary. Pay particular attention to instruction 2. **To receive credit, you must show your work.** Unexplained answers, and answers not supported by the work you show, will not receive credit.
   4. Express your answers in simplified form.

1. (25) Evaluate

   a. \( \lim_{n \to \infty} \left( 1 + \frac{1}{n} \right)^{-3n} \)

   b. \( \lim_{n \to \infty} \left( 1 - \frac{2}{n} \right)^n \)

   c. \( \int_0^\pi \cos^2 x \sin x \, dx \)

   d. The first and second derivatives of \( F(x) = \int_{\frac{2}{2}}^x \sin(t^2) \, dt \)

2. (25) A cookie box, as shown below, has been made from a 12 inch by 12 inch square of cardboard. Find the value of \( x \) that maximizes the volume of the box, and use the second derivative test to show that you have found a local maximum.
3. (25) Drawn at the left is the graph of the derivative of a mysterious function named $f$. On the right is a blank set of axes.

Graph of $\frac{df}{dx}$

a. On what interval(s) is $f$ increasing? Explain your answer.

b. On what interval(s) is $f$ decreasing? Explain your answer.

c. Tell me the critical point(s) of $f$, and tell me which are local minima and which are local maxima. Explain your answer.

d. For which value(s) of $x$ is $\frac{d^2x}{dx^2} = 0$? Explain your answer.

4. (25) Sketch the region bounded by the parabola $y = x^2 + 1$ and the line $y = -2x + 9$, and calculate the area of this region.
ANSWERS

1. a. $e^{-3}$  
   b. $e^{-2}$  
   c. $\frac{2}{3}$  
   d. $\sin(x^2), 2x\cos(x^2)$

2. $x = 2$, $V''(2) = -24 < 0$

3. a. $(\infty, 0)$ and $(2, \infty)$
   b. $(0, 2)$
   c. By the first derivative test, there is a local maximum at 0 and a local minimum at 2.
   d. -1 and 1

4. The area is 36