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Spring 07
Math 3770
Name: ______________________
Final exam
Bonetto
1. Consider the circuit shown in the figure. The system fails as soon as one of the two elements fails. Element A has a life time described by an exponential r.v. $T_A$ with parameter 1 while element B has a life time described by an exponential r.v. $T_B$ with parameter 2.

![Figure 1: Circuit](image)

(a) let $T_S$ the r.v. that describes the life time of the full system. Compute the p.d.f. of $T_S$.
   (Hint: Compute $P(T_S > t)$. Use the result to find the c.d.f. of $T_S$ and then its p.d.f.)

(b) Suppose that after a time $t$ you observe that the system has failed. Compute the probability that it was element A that failed.
2. The following data $x_i$ are the result of a random sample of size $N = 50$. They are ordered in increasing order.

$$
\begin{align*}
11.760 & \quad 14.911 & \quad 16.267 & \quad 16.316 & \quad 16.709 & \quad 16.756 & \quad 17.184 & \quad 17.195 & \quad 17.459 & \quad 17.491 \\
17.571 & \quad 17.621 & \quad 17.707 & \quad 18.211 & \quad 18.260 & \quad 18.426 & \quad 18.545 & \quad 18.729 & \quad 18.899 & \quad 19.170 \\
19.251 & \quad 19.322 & \quad 19.391 & \quad 19.500 & \quad 19.563 & \quad 19.739 & \quad 19.752 & \quad 19.761 & \quad 20.214 & \quad 20.230 \\
20.343 & \quad 20.385 & \quad 20.549 & \quad 20.802 & \quad 21.113 & \quad 21.323 & \quad 21.380 & \quad 21.524 & \quad 21.704 & \quad 21.890 \\
21.960 & \quad 22.073 & \quad 22.778 & \quad 22.918 & \quad 23.418 & \quad 23.535 & \quad 24.398 & \quad 24.650 & \quad 24.771 & \quad 26.730
\end{align*}
$$

Table 1: Data

You know that

$$
\sum_{i=1}^{50} x_i = 990.16 \quad \sum_{i=1}^{50} x_i^2 = 19983.
$$

(a) Compute the sample average and standard deviation.

(b) Compute the median and fourth spread.
(c) After finding eventual outlier, draw a box plot for the data.

(d) Choose a reasonable number of classes and draw an histogram for the data. Show your computations.
(e) Give a 98\% confidence interval for the true population mean $\mu$.

(f) How big should the sample be to obtain a precision $w$ smaller than 1. The precision $w$ is the size of the confidence interval.
3. Suppose now that you have to test the null hypothesis $H_0 : \mu = 20$ against the alternative hypothesis $H_a : \mu < 20$ using the data presented in Table 1.

(a) Test the hypothesis at 0.025 significance level.

(b) Give the P-value for the test and explain its meaning.
4. The arrival times of the next two cars at a pay booth are described by the two random variables $T_1$ and $T_2$. The j.p.d.f. of these variables is:

$$f(t_1, t_2) = \begin{cases} 
4e^{-2t_2} & t_2 > t_1 > 0 \\
0 & \text{otherwise}
\end{cases}$$

(a) Compute the marginal p.d.f. $f_{T_1}(t_1)$ and $f_{T_2}(t_2)$.

(b) Compute the conditional p.d.f. $f_{T_1|T_2}(t_1|t_2)$ and $f_{T_2|T_1}(t_2|t_1)$.

(c) **(Bonus)** Compute the j.p.d.f. of $T_1$ and $S = T_2 - T_1$. Are they independent?
5. The data reported in Table 2 are the result of a random sample from a population distributed according to the following p.d.f.:

\[
f(x; \alpha) = \begin{cases} 
\alpha x^{\alpha - 1} & x > 1 \\
0 & \text{otherwise}
\end{cases}
\]

where \( \alpha > 0 \) is a parameter to be determined.

1.9230  1.7780  1.1149  1.3711  1.5444  1.0371  3.4376

Table 2: Data

(a) Use the methods of moment to estimate \( \alpha \).

(b) Use Maximum Likelihood to estimate \( \alpha \).