Homework 1

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Additional Problems:

1. For each $k \in \mathbb{N}$ let $A_k$ be a countable set. Show that

\[ \bigcup_{k=1}^{\infty} A_k \]

is countable. In other words, a countable union of countable sets is countable.

2. Prove that the map $f : A \to B$ is a bijection if and only if there exists a map $g : B \to A$ such that $g \circ f = Id_A$ and $f \circ g = Id_B$. Show also that $g = f^{-1}$ and is uniquely determined.

3. Let $f : A \to B$ and $g : B \to C$ be bijections. Then $g \circ f$ is and bijection and $(g \circ f)^{-1} = f^{-1} \circ g^{-1}$.

4. Let $\mathcal{A} = \{A_{\alpha}\}$ be a collection of subsets of a set $S$. Prove the following two statements:

(a) $S \setminus \bigcup_{\alpha} A_{\alpha} = \bigcap_{\alpha} S \setminus A_{\alpha}$;
(b) $S \setminus \bigcap_{\alpha} A_{\alpha} = \bigcup_{\alpha} S \setminus A_{\alpha}$. 