Solution to problem 1

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**Solution.** To solve this problem we first must find an orthogonal basis for the space spanned by $x$ and $x + 1$ (because, recall, projection is defined in terms of an orthogonal basis): we can start by letting $b_1 = x$ and then to find $b_2$ we subtract the projection of $x + 1$ onto the space spanned by $x$; so,

$$b_2 = x + 1 - x \int_0^1 \frac{x^2(x + 1)dx}{\int_0^1 x^3dx} = x + 1 - x \frac{1}{4} \left(1 + \frac{1}{4} + \frac{1}{3}\right) = x + 1 - \frac{7}{3}x = -\frac{4}{3}x + 1.$$

You can get rid of denominators and negatives here – it’s perfectly legal ($b_1$ and $b_2$ will still be orthogonal) – just multiply by $-3$, and you get $b_2 = 4x - 3$.

And now, our projection is just:

$$p(x^2 + x + 1) = \frac{\langle x^2 + x + 1, x \rangle}{\langle x, x \rangle} x + \frac{\langle x^2 + x + 1, 4x - 3 \rangle}{\langle 4x - 3, 4x - 3 \rangle} (4x - 3) = \frac{47}{15} x + \frac{48 + 15 + 20 - 90}{240 - 480 + 270} (4x - 3) = \frac{47}{15} x + \frac{-7(4x - 3)}{30} = \frac{11x}{5} + \frac{7}{10}.$$