Instructions: Print your name and sign your signature to indicate that you accept the honor code. To receive full credit you must provide a proof that any answer you give is correct. You have 75 minutes to answer all the questions. Good Luck

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1) Let $f : [a, b] \to \mathbb{R}$ and suppose that there is some $M$ such that $|f'(x)| \leq M$. Prove using the definitions that $f$ is Lipschitz and continuous on $[a, b]$. 
2) Assuming that $f'$ exists on $[a, b]$ and $\lim_{x \to c} f'(x) = L$ for some $c \in (a, b)$, prove that $f'$ is continuous at $c$. 
3) Let $f : [a, b] \to \mathbb{R}$ be an integrable function with $f(x) \geq 0$ for all $x \in [a, b]$.

a) If $f$ is continuous at some $c \in (a, b)$ and $f(c) > 0$ show that

$$\int_a^b f(x) \, dx > 0.$$ 

b) If the set $C = \{x \in [a, b] : f(x) = 0\}$ has measure zero show that

$$\int_a^b f(x) \, dx > 0.$$ 

Hint: Does $[a, b]$ have measure zero?
4) Let $f : [0, 1] \rightarrow \mathbb{R}$ be the function that is 0 for all irrational numbers and $f(x) = x$ for all rational numbers. Prove that $f$ is not integrable. 

Hint: Show that the upper and lower Darboux integrals cannot be the same. Specifically show that any upper sum is bounded below by $\frac{1}{2}$. 


5) Answer the following questions True or False. Circle either T or F to indicate your answer. You do not need to justify your answer.

1. If $|f|$ is integrable on $[a, b]$ then $f$ is integrable on $[a, b]$.
   
   T   F

2. If $f$ is not integrable on $[a, b]$ then there are partitions $P$ and $Q$ of $[a, b]$ such that $L(f, Q) > U(f, P)$.
   
   T   F

3. If a function is differentiable on an open interval $I$ then it is continuous on $I$.
   
   T   F

4. Sets of measure zero must be countable.
   
   T   F

5. If a function is differentiable on an open interval $I$ then its derivative is continuous on $I$.
   
   T   F

6. If a function has bounded derivative on an interval then it is uniformly continuous on the interval.
   
   T   F

7. Every integrable function has an anti-derivative.
   
   T   F

8. The set of integrable functions form a vector space.
   
   T   F

9. The product of integrable functions is integrable.
   
   T   F

10. The composition of integrable functions is integrable.
    
    T   F