Problem 1 (8 points): Let $f : \mathbb{R} \to \mathbb{R}$. Assume $f$ is increasing. Assume $f(1) = 2$. Assume the sequence $2 + (-1)^n/n$ belongs to the image of $f$. Prove that $f$ is continuous at 1.
Problem 2 (8 points): Using the epsilon-delta definition, prove that the function \( f : \mathbb{R} \to \mathbb{R} \) given by \( f(x) = 1/(1 + x^2) \) is continuous everywhere.
Problem 3 (8 points): Is the function \( f : \mathbb{R} \to \mathbb{R}, f(x) = x^2 \) uniformly continuous? How about \( f : [0,1] \to \mathbb{R}, f(x) = x^2 \)? Justify, i.e. prove.
Problem 4 (8 points): Assume $f : \mathbb{R} \to \mathbb{R}$ is twice differentiable at $x_0$, that $f'(x_0) = 0$ and that $f''(x_0) = -2$. Prove that $x_0$ is a strict local maximum of $f$, i.e. there exist $N$, a neighborhood of $x_0$ such that $f(x) < f(x_0)$ for all $x \in N, x \neq x_0$. 
Problem 5 (8 points): Let $f_n : E \to \mathbb{R}$ be continuous functions for $1 \leq n \leq N$. Let $a_k^{(n)}$ be $N$ convergent sequences of numbers. Assume $\lim_{k \to \infty} a_k^{(n)} = a_n$. Let $f = \sum_{n=1}^{N} a_n f_n$.

True or false. Justify (prove or give counter example).

(a) $\sum_{n=1}^{N} a_k^{(n)} f_n$ converges pointwise to $f$

(b) $\sum_{n=1}^{N} a_k^{(n)} f_n$ converges uniformly to $f$

(c) $\sum_{n=1}^{N} a_k^{(n)} f_n$ converges uniformly to $f$ if $E$ is compact