Options

European call option: A contract between two parties, the writer, who issues the option, and the holder, who buys the option from the writer. The holder buys the option from the writer by paying the premium to the writer. This contract gives the right to the holder to buy a particular asset from the writer at an agreed price \( K \) at a specified time in the future \( T \). This asset is called the underlying. Frequently, but not always, the asset underlying is a stock. \( K \) is called the strike or exercise price, \( T \) is called the expiry or
Expiration date or maturity date.
European put option: the holder has now the right to sell the underlying at the price \( K \) at expiration date \( T \).
American options: like European but the right to sell or buy can be exercised at any time \( t \leq T \), i.e., at expiry or any time before.

Value of call option at expiration date \( T \) 

\[ S = \text{market price of the underlying} \]

\[ \text{(European)} \]

If \( S > K \), the at expiry \( T \), the holder can buy the underlying for \( K \) per share by exercising the option and immediately selling it for the current market price \( S \).
directly sell in the market for $S$ per share to make a profit of $S - k$ per share.

If $K > S$, it is best for the holder not to exercise the option. Thus,

$V = \text{value of the option}$

$V = V(S, t) \quad S = S(t) = \text{market price of the underlying at time } t.$

From above arguments

$V(S, T) = \begin{cases} 0 & \text{if } S \leq K \\ S - K & \text{if } S \geq K \end{cases}$

$V(S, T) = \max \{ S - K, 0 \}$,

(where $S = S(T)$). This is called the payoff function.
Value of the call option at strike

\[ V = S - K \]

Value of a put option at expiry (European)

\[ V(S,T) = \begin{cases} 
K - S & \text{if } S < K \\ 
0 & \text{if } S > K 
\end{cases} \]

\[ V = \max\{ K - S, 0 \} \]

Long position: the buying of option a call option or writing of a put, expecting the value of the underlying will go up
Short position: When we expect the value of the asset to go down, thus writing calls or buying puts.