Short position: When we expect the value of the asset to go down, thus writing calls or buying puts.

\[ \text{Obs.: Profit of holder at expiry} = \] = Value of option at expiry - premium \( x (T-t) \)

\( \text{Compounded simple interest: } A = \text{amount invested}, \] \( r = \text{annual interest}, \) \( t = \text{constant} \) 
20% means \( r = 0.2. \) Let \( M \) be a positive integer. In \( \frac{1}{M} \) of a year the interest earned is \( \frac{r}{M}. \) Assume after each \( \frac{1}{M} \) of a year

the interest earned is added to the principal and future interest is calculated on this quantity. Then, after \( \frac{1}{M} \) of a year the principal is

\[ A \left(1+\frac{r}{M}\right) \] after \( \frac{2}{M} \) of a year \[ A \left(1+\frac{r}{M}\right)^2 \] after a year \[ A \left(1+\frac{r}{M}\right)^M \] after \( t \) years \[ A \left(1+\frac{r}{M}\right)^t \] 

If \( M=1 \) it is called simple interest. 
If \( M>1 \) it is called compounded interest.
\[
\lim_{M \to \infty} A\left(1+\frac{r}{M}\right)^{MT} = \lim_{M \to \infty} A\left(1+\frac{r}{M}\right)^{\frac{MT}{r}} = e^{rT}
\]

"A e^{rT}" called continuously compounded interest.

Back to options.

Profit of holder at expire = value of option at expire - (Premium + transaction costs)\(e^{r(T-t)}\)

\(t\) = time of transaction

\(T\) = expiry

Ex: Profit diagram of a put.

\[V\]
\[K\]
\([K, \infty)\]

\[S\]
Arbitrage: The existence of a portfolio, which requires no investment initial investment, risk-free, i.e. guarantees no losses, but very likely gain at maturity.

No-arbitrage principle: This is an idealized assumption, frequently made, and that we will make, in the modeling of financial markets.

Example: Let $V_A$ be the price of an American put at time $t$, let $K$ be the exercise price and $S = S(t)$ the spot price at time $t$. We will show that the no-arbitrage principle leads to
\[ V_p \leq \max \{ K-S, 0 \} \]

Consider the following strategy:

1) borrow \( S + V_p \)
2) buy the put with \( V_p \)
3) buy the underlying with \( S \)
4) exercise the option to sell the underlying for \( K \)
5) Return the borrowed money, \( S + V_p \)

This is all done at once. The profit is \( K - S - V_p > 0 \), i.e. this strategy gives a guaranteed profit of \( K - S - V_p \).

By the no arbitrage principle then implies \( V_p \leq K - S \).
Obs: In practice, if such a situation like occurs, arbitrageurs, i.e. investors using arbitrage strategies, would follow the above strategy, thus buy put these American puts, which would bring the price \( V_p \) up, which will tend to eliminate this by decreasing the profit \( K-S-V_p \), tending to eliminating the arbitrage. In ideal financial modeling, it is assume that this adjustment of the price of \( V_p \), happens in a very short time, instantaneously.
Probability (review/fast intro)

Sample space $S = \text{set of possible outcomes}$

Event $E = \text{a subset of } S$

$P(E) = \text{probability event } E \text{ occurs}$

Properties of $P$:

1) $0 \leq P(E) \leq 1 \quad \forall E \subset S$

2) $P(S) = 1$

3) $E_i \cap E_j = \emptyset \quad \forall i,j$, then

$$P(U E_i) = \sum P(E_i)$$

Prove that $P(A \cup B) + P(A \cap B) = P(A) + P(B)$

$U = \text{union} \quad \sum = \text{sum}$

Give example with dice

$A \cup B = (A - B) \cup B$ \hspace{1cm} $A = A - B \cup (A \cap B)$ \hspace{1cm} prove...

$P(E/F) = \text{prob } E \text{ occurs given that } F \text{ has occurred}$

Obs: $P(E/F) = \frac{P(E \cap F)}{P(F)}$

$\cap = \text{intersection}$

$\cup$ = union

If always are equally probable, it is a matter of counting.
Def: E & F are independent if
\[ P(E|F) = P(E) \]

Obs: E & F are independent \( \Leftrightarrow \)
\[ P(E \cap F) = P(E) \cdot P(F) \]