Work the following problems and hand in your solutions. You may work together with other people in the class, but you must each write up your solutions independently. A subset of these will be selected for grading. Write LEGIBLY on the FRONT side of the page only, and STAPLE your pages together.

1. Problem 3.1 #2. (a) Show that if \( \nu \) is a signed measure, then
\[
E \text{ is } \nu\text{-null } \iff |\nu|(E) = 0.
\]
(b) Show that if \( \mu, \nu \) are signed measures, then
\[
\nu \perp \mu \iff |\nu| \perp \mu \iff \nu^+, \nu^- \perp \mu.
\]

2. Problem 3.1 #3. Let \( \nu \) be a signed measure on \((X, \mathcal{M})\).
(a) Show that \( L^1(\nu) = L^1(|\nu|) \).
(b) Show that if \( f \in L^1(\nu) \), then \( |\int f \, d\nu| \leq \int |f| \, d|\nu| \).
(c) Show that if \( E \in \mathcal{M} \), then
\[
|\nu|(E) = \sup \left\{ \left| \int_E f \, d\nu \right| : |f| \leq 1 \right\}.
\]

3. Problem 3.1 #7. Let \( \nu \) be a signed measure on \((X, \mathcal{M})\), and choose \( E \in \mathcal{M} \). Show that
\[
\nu^+(E) = \sup \left\{ \nu(A) : A \in \mathcal{M}, A \subseteq E \right\},
\]
\[
\nu^-(E) = -\inf \left\{ \nu(A) : A \in \mathcal{M}, A \subseteq E \right\},
\]
\[
|\nu|(E) = \sup \left\{ \sum_{k=1}^n |\nu(E_k)| : n \in \mathbb{N}, E_k \in \mathcal{M}, E = \bigcup_{k=1}^n E_k \text{ disjointly} \right\}.
\]