Names: ________________________________

Solutions are to be written on the board.

1. Give an example, and justify your answer, of a function from \( \mathbb{N} \) to \( \mathbb{N} \) that is
   (a) one-to-one but not onto.
   (b) onto but not one-to-one.
   (c) both onto and one-to-one but not the identity function.
   (d) neither onto nor one-to-one.

2. Let \( A = [0, 1] \) denote the closed interval of real numbers \( x \in \mathbb{R} \) such that \( 0 \leq x \leq 1 \). Give an example of two different bijective functions \( f_1 \) and \( f_2 \) from \( A \) to \( A \), neither of which is the identity function. Justify your answers.

3. Let \( X, Y, \) and \( Z \) be sets with \( A, B \subseteq X \) and \( C, D \subseteq Y \). Let \( f : X \to Y \) and \( g : Y \to Z \) be functions. Prove the following statements.
   (a) If \( A \subseteq B \), then \( f(A) \subseteq f(B) \).
   (b) If \( C \subseteq D \), then \( f^{-1}(C) \subseteq f^{-1}(D) \).

4. Let \( f : A \to B \) and \( g : B \to C \) be functions on nonempty sets \( A, B, \) and \( C \). Prove that if \( f \) and \( g \) are bijective, then so is \( g \circ f \) directly from the definitions.

5. Prove that a function \( f : A \to B \) is a bijection if and only if there exists \( g : B \to A \) with \( g \circ f = i_A \) and \( f \circ g = i_B \).

6. Let \( A \) and \( B \) be finite nonempty sets with \( |A| = |B| \), and let \( f \) be a function from \( A \) to \( B \). Prove that \( f \) is one-to-one if and only if \( f \) is onto.