1. Fix an integer \( n \geq 2 \). Disprove: For all \( a, b, c \in \mathbb{Z} \), if \( ac \equiv bc \pmod{n} \), then \( a \equiv b \pmod{n} \).

*In modular arithmetic, addition, subtraction, and multiplication behave as one would expect. However, there is NO general division rule!*

2. Suppose that \( R \) and \( S \) are relations on the nonempty set \( A \). Prove or disprove:

   (a) If \( R \cap S \) is reflexive, then so are \( R \) and \( S \).
   
   (b) If \( R \cap S \) is symmetric, then so are \( R \) and \( S \).
   
   (c) If \( R \cap S \) is transitive, then so are \( R \) and \( S \).

3. (a) Let \( R \) be the relation defined on \( \mathbb{Z} \) by \( a R b \) if \( a + b \) is even. Show that \( R \) is an equivalence relation and determine the distinct equivalence classes.
   
   (b) Consider the relation \( R' \) where “even” is replaced by “odd” above. Which properties of an equivalence relation does \( R' \) possess?

4. Let \( R \) be the relation defined on \( \mathbb{Z} \) by \( a R b \) if and only if \( a + b \equiv 0 \pmod{3} \). Show that \( R \) is not an equivalence relation.

5. Let \( S \) be a nonempty subset of \( \mathbb{Z} \) and let \( R \) be a relation defined on \( S \) by \( a R b \) if and only if \( (a + 2b) \equiv 0 \pmod{3} \).
   
   (a) Prove that \( R \) is an equivalence relation.
   
   (b) If \( S = \{-7, -6, -2, 0, 1, 4, 5, 7\} \), then what are the distinct equivalence classes?
   
   (c) What are the distinct equivalence classes when \( S = \mathbb{Z} \)?

6. A relation \( R \) is defined on \( \mathbb{Z} \) by \( a R b \) if and only if \( 3a + 5b \equiv 0 \pmod{8} \). Prove that \( R \) is an equivalence relation.

7. Try to generalize the problems above. Fix integers \( x, y \), and \( n \geq 2 \). Let \( R \) be the relation on \( \mathbb{Z} \) defined by \( ax + by \equiv 0 \pmod{n} \). Can you find a condition on \( x \) and \( y \) such that \( R \) is always an equivalence relation? Justify your answer.