ICA 11

February 13th

Names:______________________________

Solutions are to be written on the board.

1. Let $f: \mathbb{Z}_5 \to \mathbb{Z}_5$ be a function defined by $f([a]) = [2a + 3]$.
   (a) Show that $f$ is well-defined.
   (b) Determine whether $f$ is bijective.

2. Prove or disprove: For every nonempty set $A$, there exists an injective function $f: A \to \mathcal{P}(A)$.

3. Let $f$, $g$, and $h$ be functions with the given domains and co-domains.
   (a) Suppose that $g: T \to U$, $h: T \to U$, and $f: S \to T$ is onto. Prove that if $g \circ f = h \circ f$, then $g = h$.
   (b) Suppose that $g: S \to T$, $h: S \to T$, and $f: T \to U$ is 1-1. Prove that if $f \circ g = f \circ h$, then $g = h$.

4. Let $A$ and $B$ be nonempty sets. Prove that if $f: A \to B$, then $f \circ i_A = f$ and $i_B \circ f = f$.

5. Let $f: A \to B$, $g: B \to C$, and $h: C \to D$ be functions on nonempty sets $A$, $B$, $C$, and $D$. Prove that function composition is associative, that is prove that $h \circ (g \circ f) = (h \circ g) \circ f$.

6. Let $S$ be a subset of a universal set $U$. The characteristic function $f_S$ of $S$ is the function from $U$ to the set $\{0, 1\}$ such that $f_S(x) = 1$ if $x \in S$ and $f_S(x) = 0$ if $x \notin S$.
   Let $A$ and $B$ be subsets of $U$. Show that for all $x$
   (a) $f_{A \cap B}(x) = f_A(x) \cdot f_B(x)$
   (b) $f_{A \cup B}(x) = f_A(x) + f_B(x) - f_A(x) \cdot f_B(x)$
   (c) $f_{\overline{A}}(x) = 1 - f_A(x)$