Math 1501: §4.5: Max Min Problems
If $f(x) = (a-x)(b-x)$, then on the interval $[a, b]$ this is a parabola, equal to zero at $x = a, b$. It is positive on $(a, b)$, and its maximum occurs at the midpoint $x = (a + b)/2$. Its maximum value is $(b - a/2)^2$. 
Example

Find the dimensions of a rectangle of maximum area that can be inscribed with the base of the rectangle on the short side of an isosceles triangle of base 6 units and height 12 units.
Example

Fix $0 < a < b < \infty$. Maximize the area of a rectangle with side lengths $x$ and $y$ subject to the condition that $ax + by = 1$. 
**Example**

Find the right cylinder of volume 22 units$^3$, with least surface area.
Example

Find the dimensions of a square and a circle of total perimeter one, and minimal area. What about maximal area?

$$2\pi r + 4s = 1$$
Example

Find the dimensions of a right cylinder of maximal volume inscribed in a sphere of radius one.
Example

A window in the shape of a rectangle capped by a semicircle is to have a perimeter $p$. Choose the radius of the semicircular part so that the window admits the most light.
**Example**

The highway department is asked to construct a road between points $A$ and point $B$. Point $A$ lies on an abandoned road that runs east-west. Point $B$ is 3 miles north of the point on the old road that is 5 miles east of $A$. The road will be constructed by using a portion of the old road. Restoring the old road costs $2$ million per mile, and the new road costs $4$ million per mile. How much of the old road should be restored to minimize the cost of the project.
Example

Find the dimensions of a right circular cone of maximal volume that can be inscribed in a sphere of radius 1.
§4.6: Concavity

**Definition**
A function is concave up on an interval \( I \) iff \( f' \) increases on \( I \); it is concave down iff \( f' \) decreases on \( I \).

**Definition**
A point \( c \) is a point of inflection for function \( f \) iff the function changes concavity at \( c \). That is there is a \( \delta > 0 \) so that \( f \) is concave, of different signs, on the two intervals \((c - \delta)\) and \( c + \delta \).

**Theorem**
If \( f \) has two derivatives and if for some interval \( I \), \( f''(x) > 0 \) for \( x \in I \), then \( f \) is concave up on the interval \( I \).
Example

Determine the concavity of the function \( f(x) = \frac{x^2}{4} - \sin x \).