This test is to be taken without calculators and notes of any sorts. The allowed time is 50 minutes. Provide exact answers; not decimal approximations! For example, if you mean $\sqrt{2}$ do not write 1.414….. Show your work, otherwise credit cannot be given. 
Write your name, your section number as well as the name of your TA on EVERY PAGE of this test. This is very important.
I: (25 points) Decide whether the following series converge or diverge: State which kind of convergence test you are using.

a) \[ \sum_{k=0}^{\infty} \frac{(3k)!k!}{[(2k)!]^2} \]

b) \[ \sum_{k=2}^{\infty} \log \left( 1 - \frac{1}{k^2} \right) \]

c) \[ \sum_{k=2}^{\infty} \frac{1}{k[\log k]^2} \]
II: (25 points) a) Find the Taylor series for the function

\[ f(x) = \int_0^x e^{-y^2} \, dy \]

Find a polynomial that approximates \( f(x) \) on the interval \([0, 1]\) with an error less than \(10^{-3}\).

b) Find the Taylor series of the function

\[ \frac{1}{4 - 3x} \]

c) Sum the series

\[ \sum_{k=1}^{\infty} (-1)^k k \left( \frac{3}{4} \right)^k \]
III: (25 points) a) Find the interval of convergence of the power series

a) \[ \sum_{k=1}^{\infty} \frac{1}{k} (x - 2)^k 2^{-k} \]

b) \[ \sum_{k=1}^{\infty} \frac{[\log(k)]^k}{k!} x^k \]

c) \[ \sum_{k=2}^{\infty} \frac{\log k}{k^2} x^k \]
IV: (25 points) Solve the initial value problems
   a) \( y'' - 2y' + 5y = 0 \), \( y(0) = 0 \), \( y'(0) = 1 \).
   b) \( y' = x(1 + y^2) \), \( y\left(\frac{\pi}{2}\right) = 0 \).
   c) \( y' + 3xy = x \), \( y(0) = 1 \)