Test 2 for Calculus II, Math 1502 G1-G5, October 5, 2010

Name:

Section:

Name of TA:

This test is to be taken without calculators and notes of any sorts. The allowed time is 50 minutes. Provide exact answers; not decimal approximations! For example, if you mean $\sqrt{2}$ do not write 1.414... Show your work, otherwise credit cannot be given.

Write your name, your section number as well as the name of your TA on EVERY PAGE of this test. This is very important.
I: Decide whether the following series converge or diverge. State which convergence test you are going to use.

a) (8 points)
\[ \sum_{k=0}^{\infty} \frac{[k!]^2}{(3k)!} \]

b) (8 points)
\[ \sum_{k=1}^{\infty} \frac{3k^2}{k!} \]

c) (9 points)
\[ \sum_{k=1}^{\infty} k^{-(1+\frac{1}{k})} \]
II: a) (9 points) Consider the alternating series
\[ L = \sum_{k=0}^{\infty} (-1)^k 10^{-k^2} \]
Find the smallest value of \( N \) so that the \( N \)-th partial sum \( s_N \) satisfies \( |L - s_N| < 10^{-15} \).

b) (8 points) Find the power series expansion for \( \cosh x := \frac{1}{2} (e^x + e^{-x}) \).

c) (8 points) Sum the series
\[ \sum_{k=0}^{\infty} (k + 2)2^{-k} \]
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III: Find the interval of convergence of the following power series. State which convergence test you are going to use for computing the radius of convergence.

a) (8 points)

\[
\sum_{k=0}^{\infty} \frac{\sqrt{k!}}{k^k} x^k
\]

b) (9 points)

\[
\sum_{k=1}^{\infty} (-1)^k \frac{1}{k} \left(\frac{x+3}{2}\right)^k
\]

c) (8 points)

\[
\sum_{k=1}^{\infty} \left(1 + \frac{1}{k}\right)^{-k} (x-1)^k
\]
IV: a) (12 points) Solve the initial value problem

\[ y'' + y' - 2y = 0 , \quad y(0) = 0 , \quad y'(0) = 1 . \]

b) (13 points) At a certain moment, a tank contains 100 liters of brine with a concentration 40 grams of salt per liter. The brine is continuously drawn off at a rate of 10 liters per minute and replaced by brine containing 20 grams salt per liter. Find the amount of salt in the tank at time \( t \) later.